

# III.D FOURIER IMAGE PROCESSING TECHNIQUES

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### Outline

Optical vs. Computer Image Analysis/Processing

Digital Processing Steps

Hardware / Software

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.a Comparison of Optical and Computer Image Analysis

#### **Advantages of Computer Analysis :**

Several advantages to processing images by digital rather than optical Fourier methods

Main advantages derive from the **quantitative** nature and **virtual infinite flexibility** of data manipulation

1. **Example:** In "**pseudo**" **optical filtering** (digital equivalent of OF), filter masks can be designed with an infinite variety and combination of hole sizes, shapes and "transparencies"
2. **3D reconstruction** and **rotational filtering** are impractical or impossible using the optical Fourier techniques
3. **Quantitative analysis or manipulation** of data **not practical** by optical means, but is the essence of computational processing

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.a Comparison of Optical and Computer Image Analysis

3. **Quantitative analysis or manipulation** of data **not practical** by optical means, but is the essence of computational processing

#### **Examples:**

- Removal of **image aberrations** (*e.g.* astigmatism; defocus)
- Removal of **specimen distortions** (*e.g.* filament curvature)
- Averaging of **separate** 2D or 3D reconstructions

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.a Comparison of Optical and Computer Image Analysis

#### Disadvantages of Computer Analysis:

- Necessity for **discrete sampling** of data

Introduces **aliasing artifacts** (transform overlap) which can be reduced by judicious choice of scanning conditions, but **never totally removed**

- Cost: may be prohibitive

No sense developing a system whose main purpose is to provide **qualitative** examination of specimen OD patterns (Optical diffractometers are cheap and operate at speed of light)

- OD still provides best method for **screening images** (quick and inexpensive)

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.a Comparison of Optical and Computer Image Analysis

#### OPTICAL

Original micrograph used

OD bench can be simple and inexpensive

Formation of diffraction pattern  
instantaneous

Filtering operations require high quality  
(*i.e.* expensive) optics

Accurate filter masks tedious to make

Filtered image recorded photographically

Quantitative information difficult or nearly  
impossible to obtain

Amplitudes and phases difficult to  
manipulate

#### COMPUTER

Micrograph digitized and "floated"

Fast computer needed for "interactive" results

Careful digitization normally slow and compu-  
tation of diffraction patterns may take several  
seconds

Computers get more powerful and cheaper  
every day

Only limited by quality of software

Reconstructed images displayed and photo-  
graphed using computer graphics devices

Essence of computing IS to be quantitative

Infinite control over amplitudes and phases

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.a Comparison of Optical and Computer Image Analysis

#### OPTICAL

Attenuation of zero-order beam to improve contrast in filtered image (may cause frequency doubling)

Imposing idealized, **non**-translational symmetries virtually impossible

Correction for lattice distortion virtually impossible

Data (ODs and filtered images) are continuous (*i.e.* vary smoothly)

Fast for screening and selecting best images for additional analysis

Reconstruction of 3D structure essentially impossible

Impractical to average data from different micrographs

#### COMPUTER

Control of contrast simple and straightforward

Any symmetries (even incorrect) can be easily imposed

Lattice distortions can be corrected (reinterpolate original image onto perfect lattice)

Data are discrete (pixels)

Not until CCD technology gets cheap

Procedures rather straightforward with "right" software

Easy to average data from different micrographs

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### **Typical digital processing procedure includes:**

1. Selection of images
2. Densitometry
3. Boxing and floating the digital image
4. Fourier transformation
5. Indexing of two-dimensional lattices
6. 2D filtering/3D reconstruction (back-transformation)

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## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 1. Image Selection

Micrographs are examined by eye and/or by OD to select a subset of 'best' images for digital processing

- OD pattern from carbon support film provides **rapid check on microscope CTF conditions** at the time the micrograph was recorded (*i.e.* defocus level, astigmatism, drift or vibrations, etc.)
- OD is generally unsuitable for selecting **individual particles** for digital, rotational filtering

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 2. Densitometry

#### **Goal:**

Convert **optical densities** in the photographic emulsion to a digital image (a **numerical array** corresponding to the relative optical densities in the image)

Each density value in the digitized image is represented as a **pixel** with an intensity ranging between 0 and 255 (an eight bit number) or 4096 (12-bit number) or even higher in some CCD cameras

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 2. Densitometry

Information content in a **single 1024 by 1024** digital image (1,048,576 pixels) is quite staggering: more than the text portion of the lecture notes for both BIO 595R and 595W!

**NOTE:** at a raster step size of **7  $\mu\text{m}$**  (smallest step size on Zeiss scanner), the area of the micrograph digitized for a 1024 by 1024 array would be  $\sim 50 \text{ mm}^2$  or **0.625%** (1/160th) of an 8 x 10 cm micrograph

Hence, the information content of **one** micrograph digitized at 7  $\mu\text{m}$  is about **160 times** the entire course contents!!!

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 2. Densitometry

#### **Rule of Thumb:**

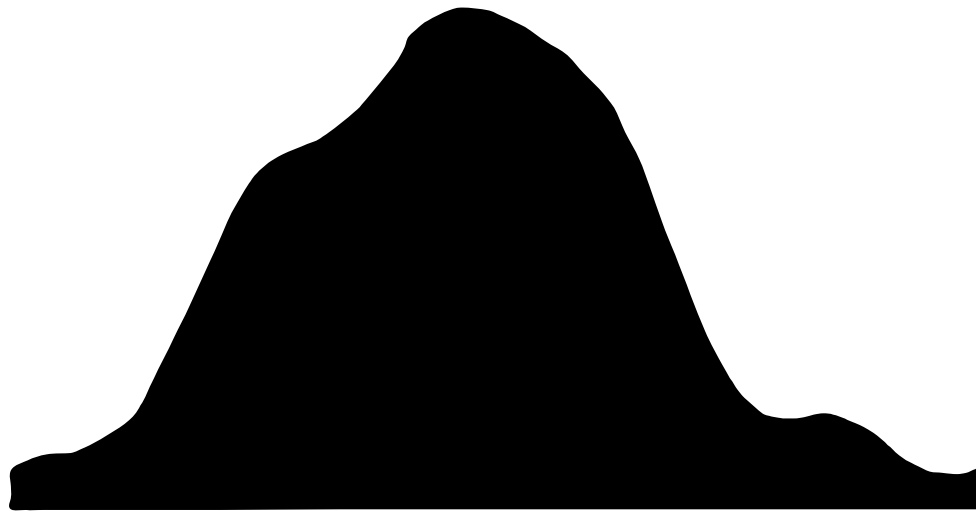
Images should be scanned to give pixels of a size corresponding to **ONE-THIRD OR LESS** than the expected resolution in the image in order to minimize **aliasing** artifacts

This condition is referred to as **over-sampling** the data

Data **under-sampling** leads to **loss** of resolution

### III.D.3.b Digital Processing Steps

#### 2. Densitometry



**Continuous**

### III.D.3.b Digital Processing Steps

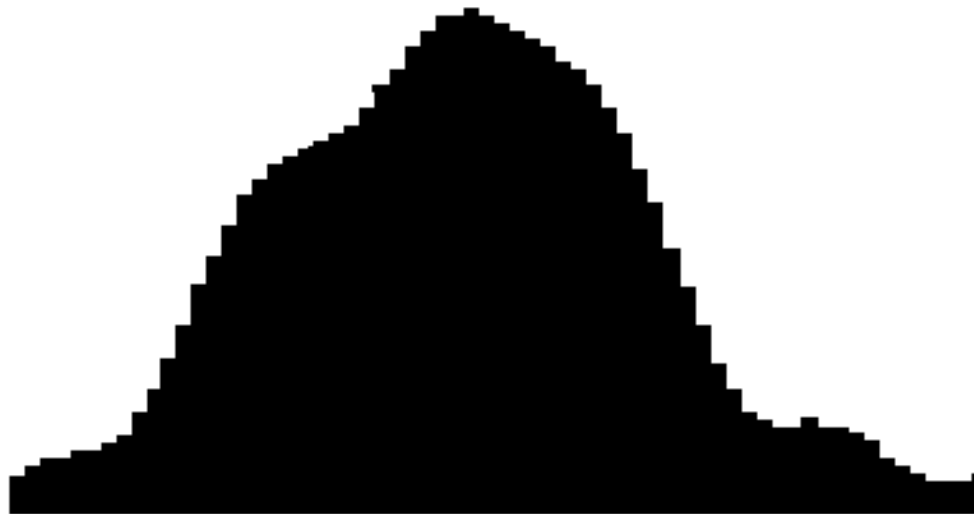
#### 2. Densitometry



128

### III.D.3.b Digital Processing Steps

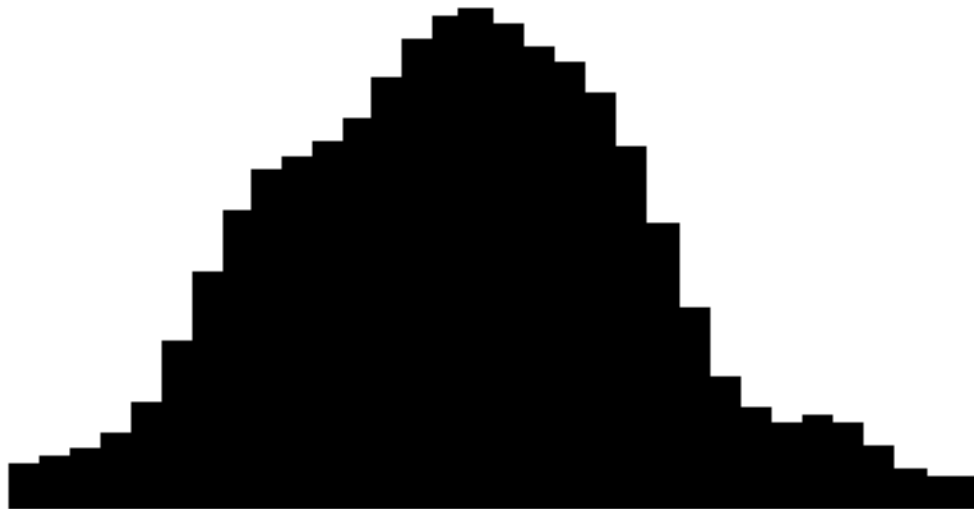
#### 2. Densitometry



64

### III.D.3.b Digital Processing Steps

#### 2. Densitometry

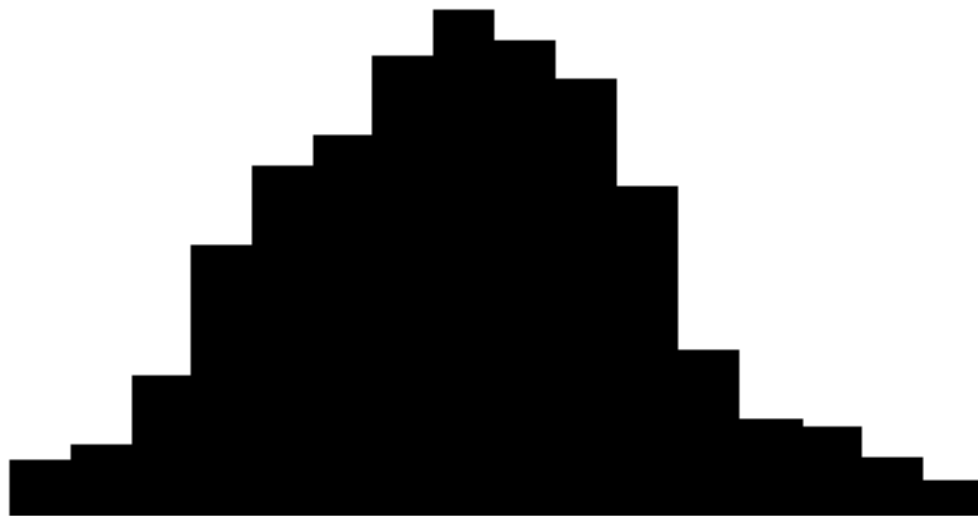


**32**



### III.D.3.b Digital Processing Steps

#### 2. Densitometry



16

### III.D.3.b Digital Processing Steps

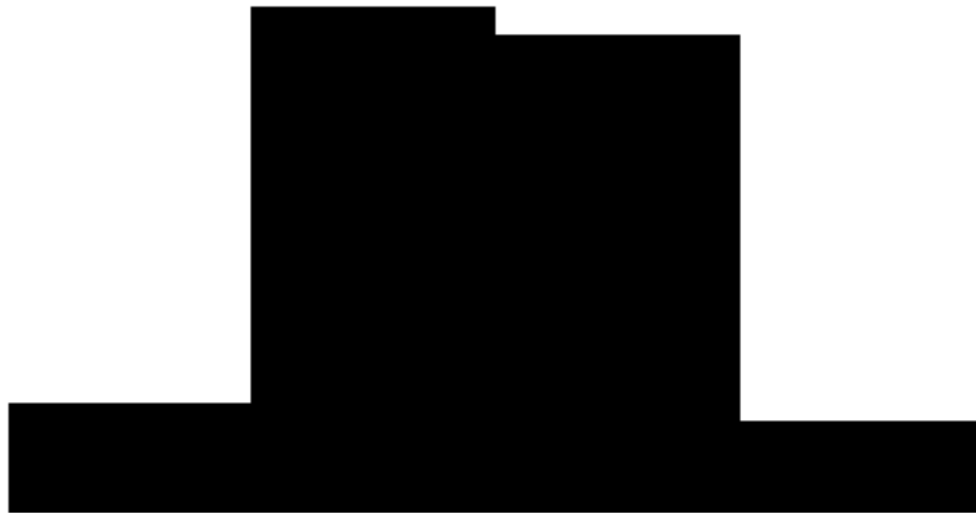
#### 2. Densitometry



8

### III.D.3.b Digital Processing Steps

#### 2. Densitometry



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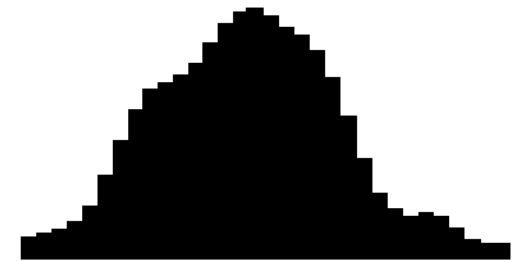
#### 2. Densitometry



128



64



32



4



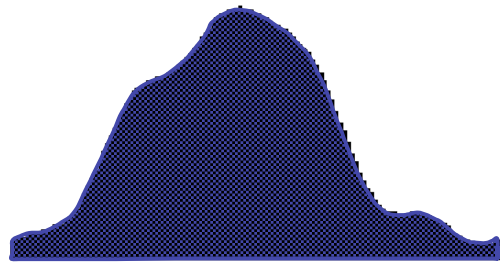
8



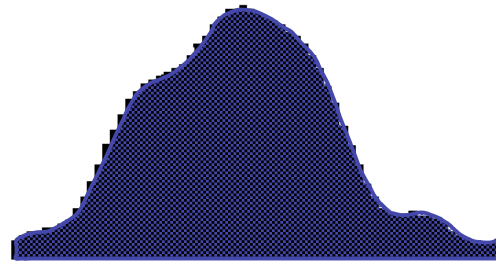
16

### III.D.3.b Digital Processing Steps

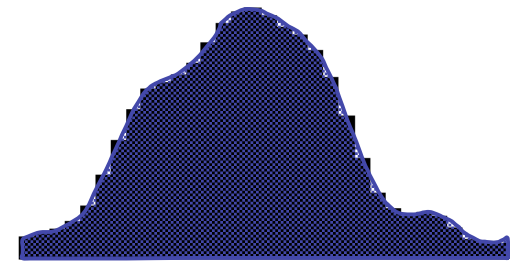
## 2. Densitometry



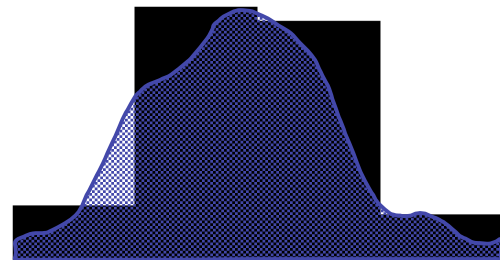
128



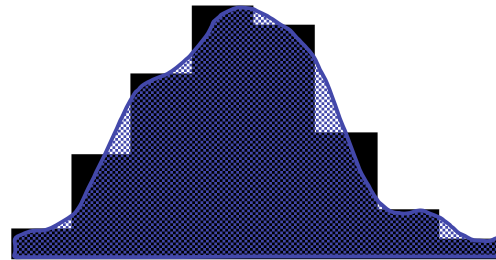
64



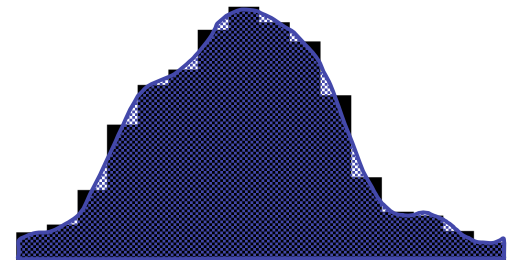
32



4



8

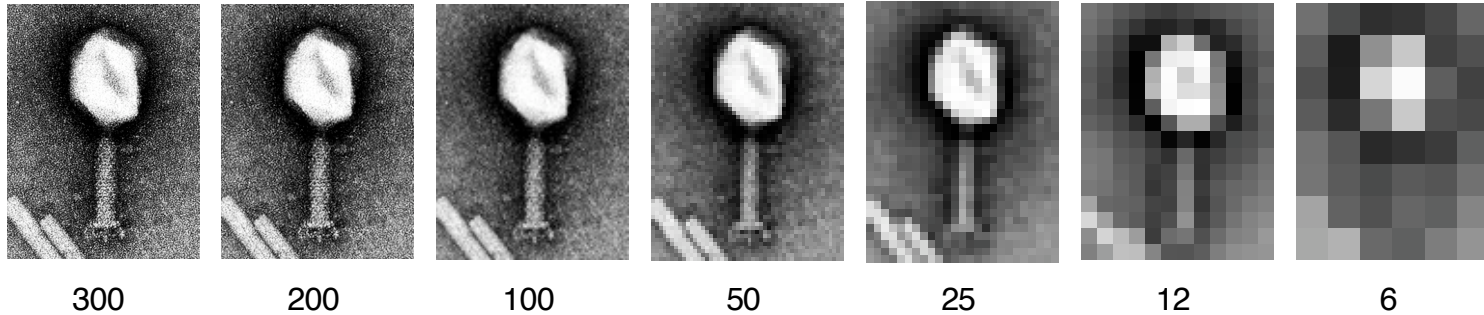


16

### III.D.3.b Digital Processing Steps

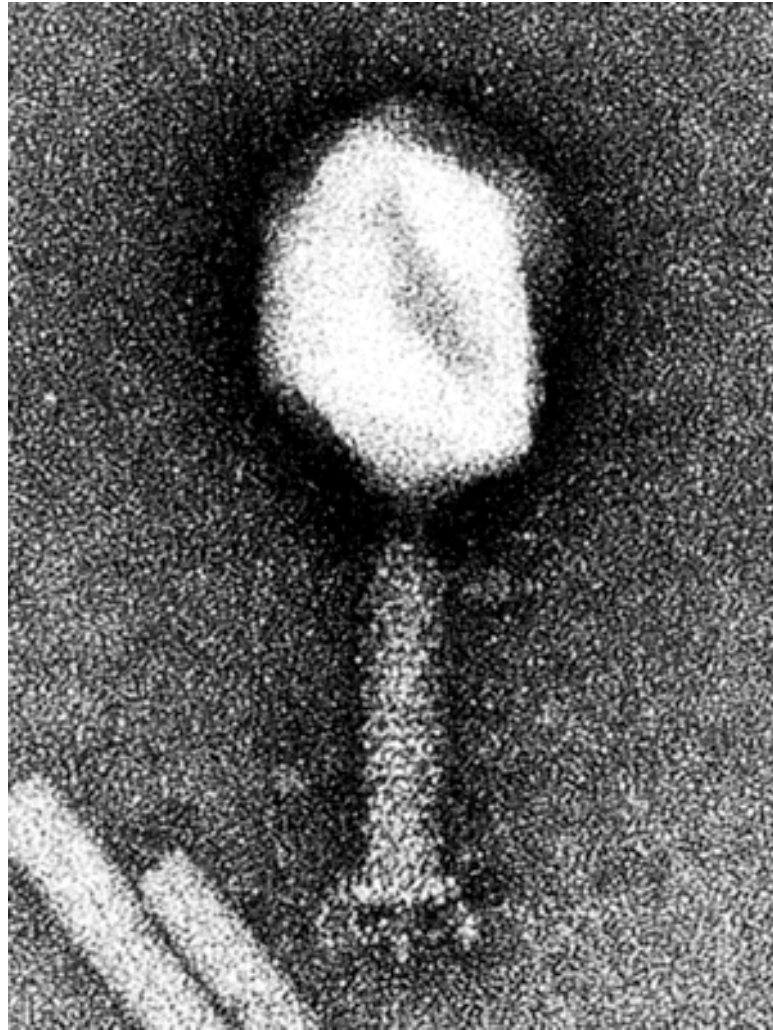
#### 2. Densitometry

Negatively-stained T4 bacteriophage sampled at different pixel resolutions (in dpi)



### III.D.3.b Digital Processing Steps

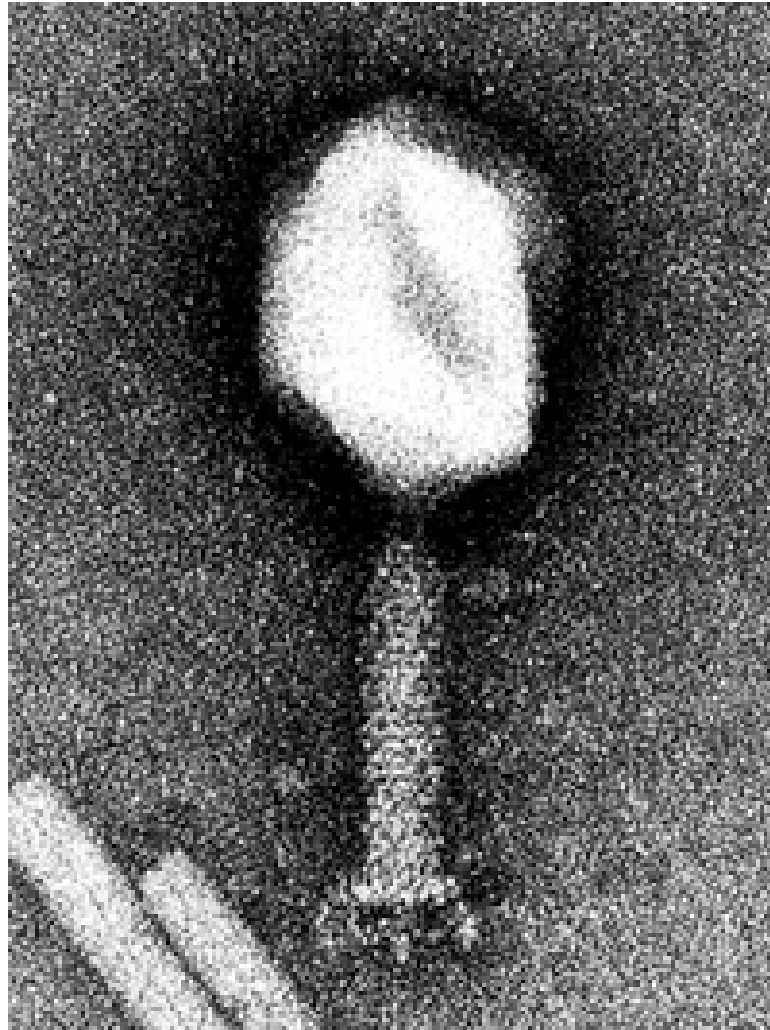
#### 2. Densitometry



300

### III.D.3.b Digital Processing Steps

#### 2. Densitometry



200



### III.D.3.b Digital Processing Steps

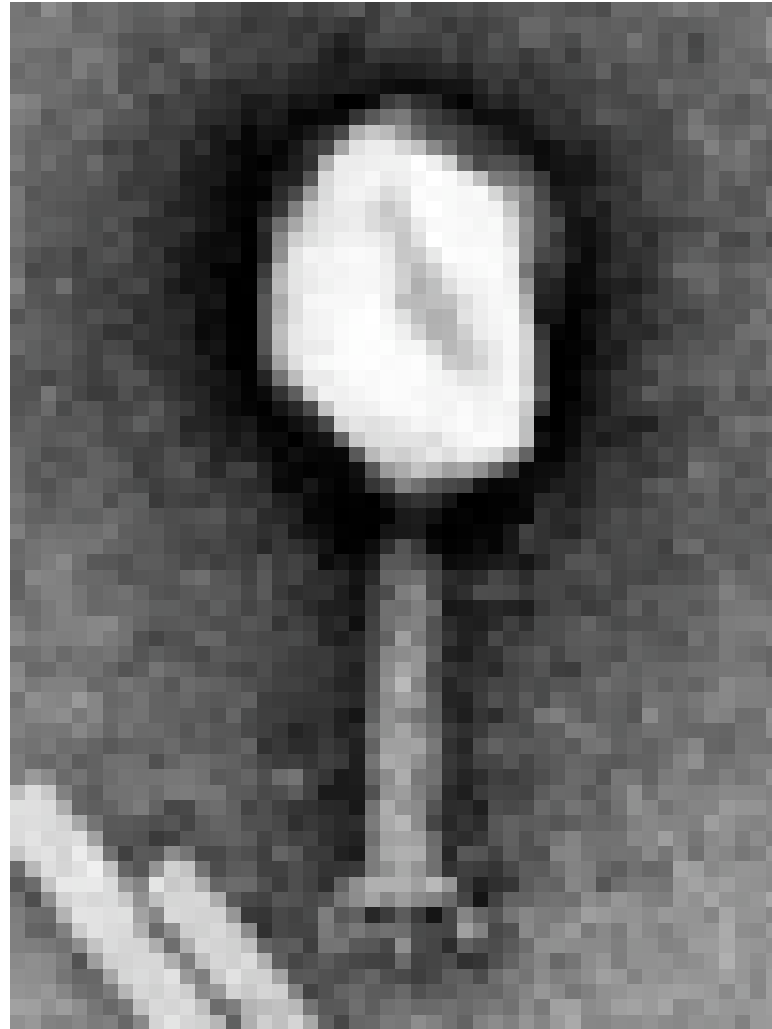
#### 2. Densitometry



100

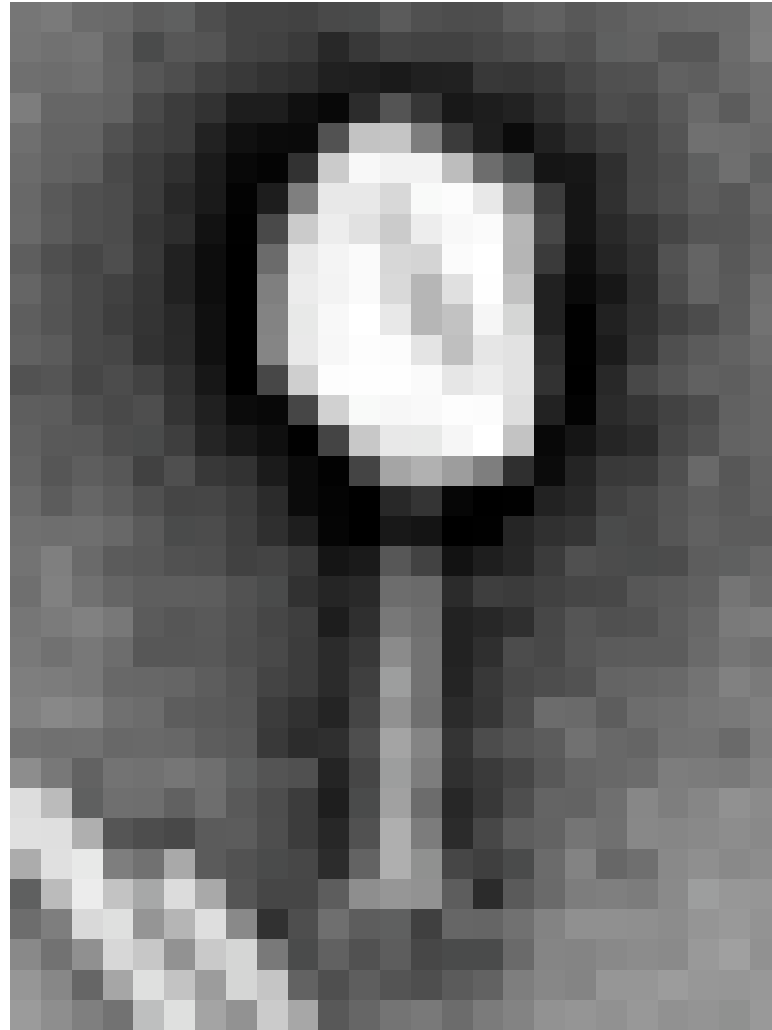
### III.D.3.b Digital Processing Steps

#### 2. Densitometry



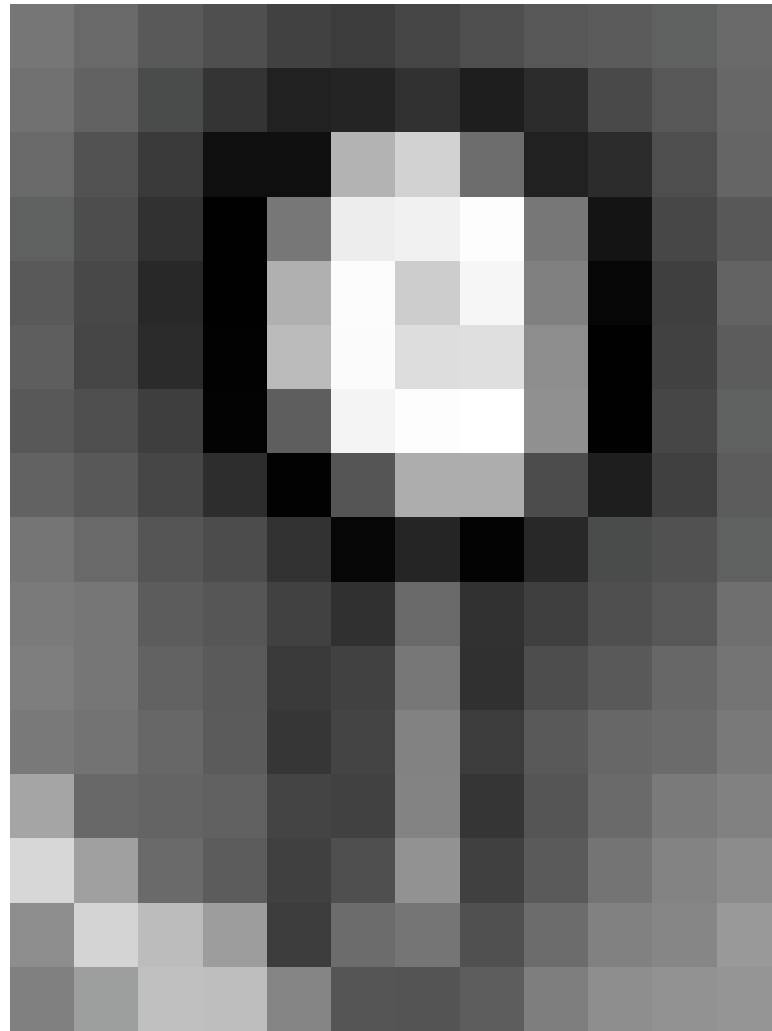
### III.D.3.b Digital Processing Steps

#### 2. Densitometry



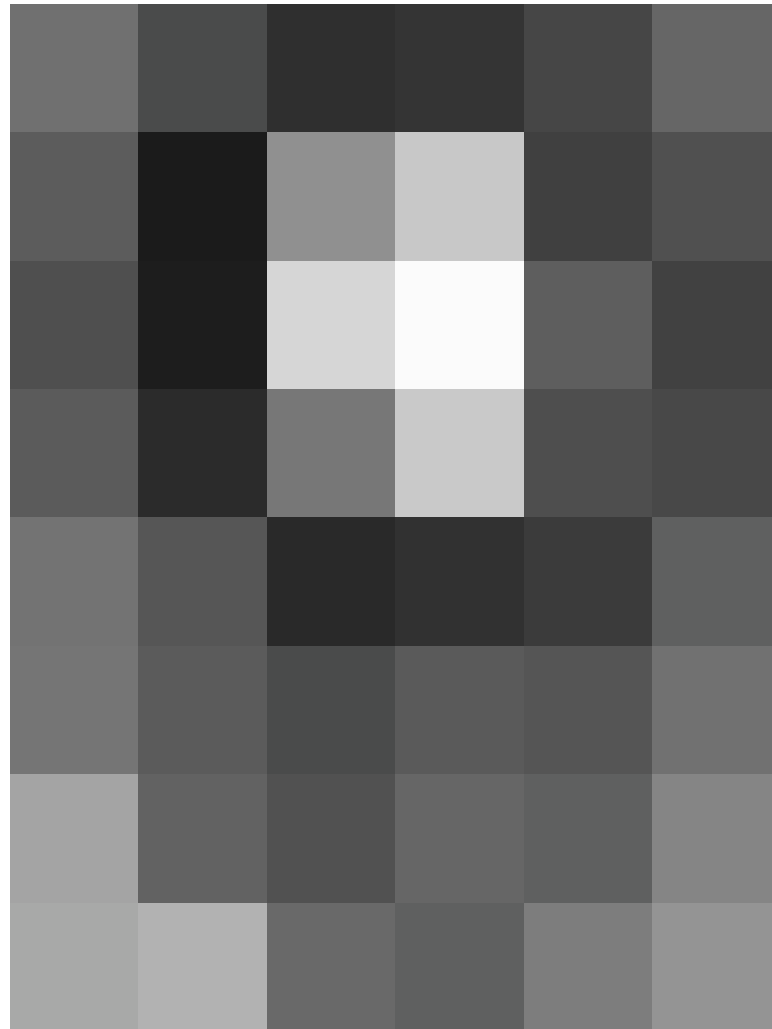
### III.D.3.b Digital Processing Steps

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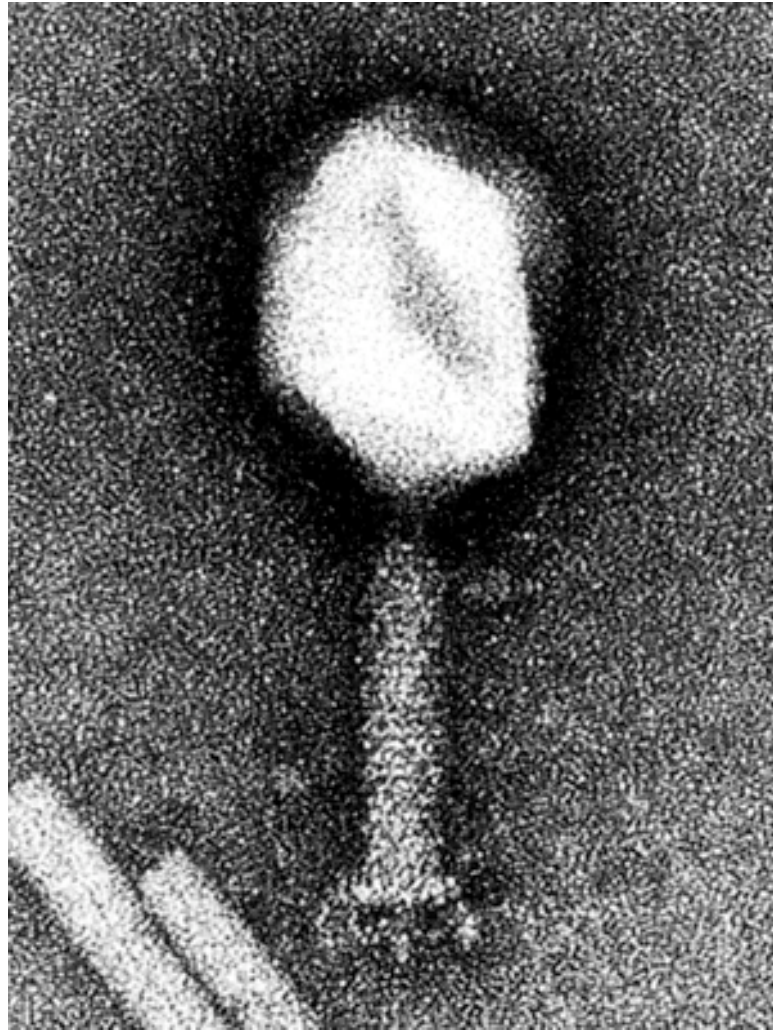
### III.D.3.b Digital Processing Steps

#### 2. Densitometry



### III.D.3.b Digital Processing Steps

#### 2. Densitometry



300

### III.D.3.b Digital Processing Steps

#### 2. Densitometry

#### **Data Over-Sampling: Good News / Bad News**

##### **Good News:**

- No loss of resolution recorded in the micrograph

##### **Bad News:**

- Increased computation owing to increased data

### III.D.3 Digital Fourier Analysis of Electron Micrographs

#### III.D.3.b Digital Processing Steps

##### 2. Densitometry

Step size (pixel resolution) in the biological specimen depends on magnification of the micrograph scanned

**Example:** Micrograph magnification = **45,000X**

Scan raster size = **14  $\mu\text{m}$**

Each pixel corresponds to **0.311 nm** at specimen

Thus, based on the scanning *Rule of Thumb*, at best can only recover information out to  $\sim 0.933$  nm (= 3 x 0.311)

**Note:** Calculation assumes the specimen is **preserved** to this resolution and the **electron optical conditions** allow recovery of this information



### III.D.3 Digital Fourier Analysis of Electron Micrographs

#### III.D.3.b Digital Processing Steps

#### 2. Densitometry

Table of **nominal pixel sizes** (in nm) recoverable from a digitized image for different scanner step sizes:

#### Nominal Pixel Size (nm)

FEI CM300 MICROGRAPH MAG	ZEISS PHODIS SCAN STEP SIZE ( $\mu\text{m}$ )			
	7	14	28	56
13,500	0.519	1.037	2.074	4.148
19,500	0.359	0.718	1.436	2.872
24,000	0.292	0.583	1.167	2.333
33,000	0.212	0.424	0.848	1.697
45,000	0.156	0.311	0.622	1.244
61,000	0.115	0.230	0.459	0.918

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#### III.D.3.b Digital Processing Steps

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**Note:** **Actual** pixel size must be determined from calibrated microscope magnifications

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 2. Densitometry

#### **Scanning Rule of Thumb #1:**

Scan images at raster settings corresponding to **ONE-THIRD OR LESS** than the expected resolution in the image in order to minimize aliasing artifacts

#### **Scanning Rule of Thumb #2:**

Generally best to scan images and store them with the **smallest** nominal pixel size (*e.g.* 7  $\mu\text{m}$  on Zeiss)

Subsequent processing can be performed if desired with larger size pixels by **reinterpolating** or **binning** the original scanned image (the 'gold' standard)

# III.D.3 Digital Fourier Analysis of Electron Micrographs

## III.D.3.b Digital Processing Steps

### 2. Densitometry

#### KEY CONCEPT:

Always wise to **carefully plan** out your experiments

- Take a best guess at the resolution you might expect to achieve in your images
- Divide this value by 3 and choose a magnification appropriate for the scanner step size you select
- If radiation damage is a problem (always is!), opt for the **smallest step size and lowest magnification**

FEI CM300 MICROGRAPH MAG	ZEISS PHODIS SCAN STEP SIZE ( $\mu\text{m}$ )			
	7	14	28	56
13,500	0.519	1.037	2.074	4.148
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**Nominal Pixel  
Size (nm)**

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 2. Densitometry

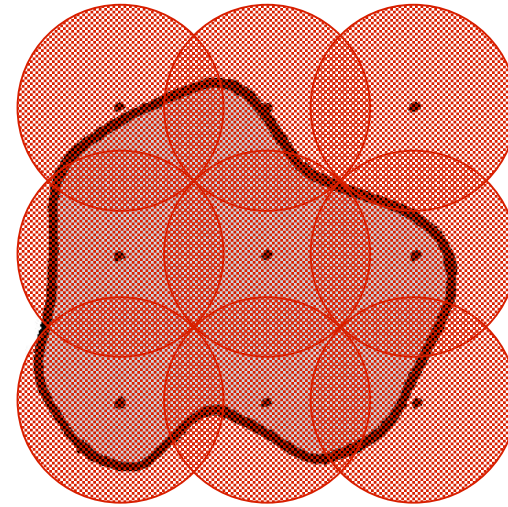
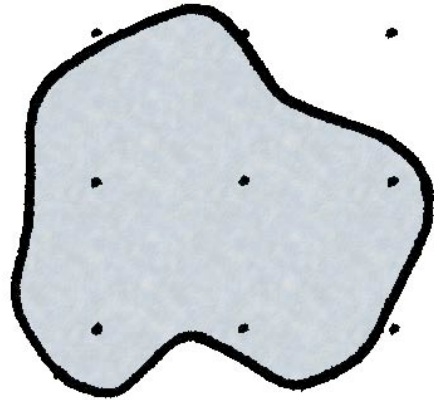
Microdensitometers are computer-driven devices

**Measure optical densities** in micrograph on a **square grid** pattern (*i.e.* at equal step sizes in two mutually perpendicular directions)

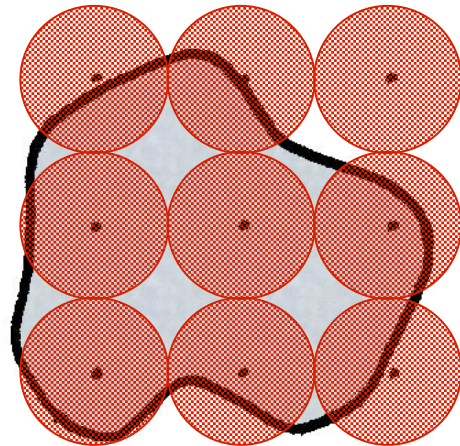
Transmission of small beam of light passing through micrograph is measured with a photomultiplier or CCD camera which **converts analog signal** (beam of light) **to a digital signal** (number ranging between 0 and 255 or higher)

### III.D.3.b Digital Processing Steps

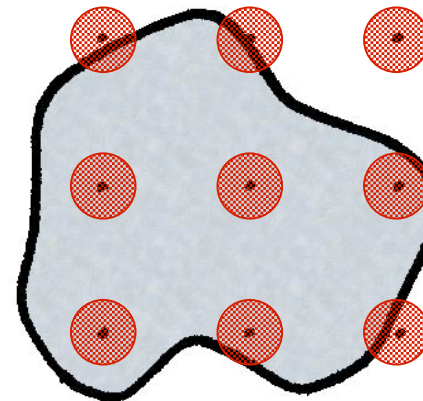
#### 2. Densitometry



Data smearing



No smearing;  
Minimal data loss

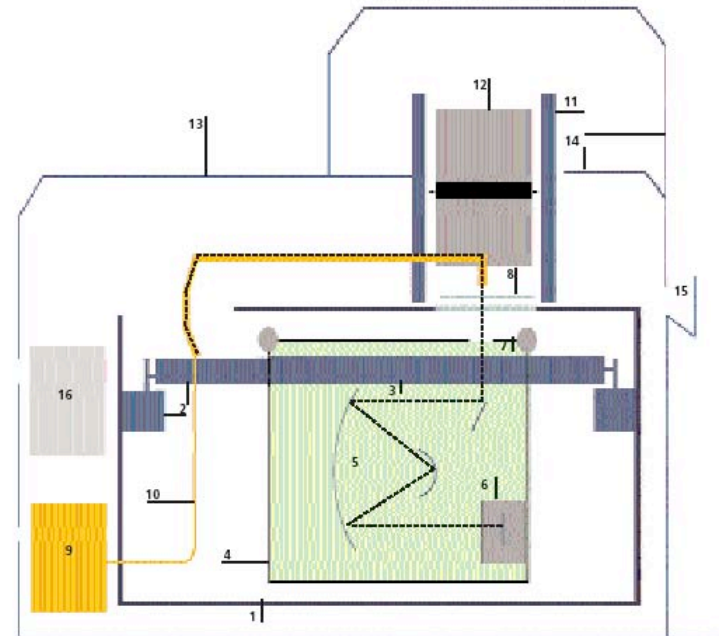


No smearing;  
Substantial data loss

# III.D.3 Digital Fourier Analysis of Electron Micrographs

## III.D.3.b Digital Processing Steps

### 2. Densitometry



- (1) Cast-iron enclosure
- (2) Primary guideway and primary carriage with (3) linear encoder
- (4) Secondary carriage with (5) mirror lens and (6) CCD module
- (7) Photo stage for scan copy and (8) glass cover plate
- (9) Lamp module with (10) fiber glass optics
- (11) Autowinder with (12) film roll (14) instrument enclosure with cover and (15) control panel
- (16) Electronics module

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 2. Densitometry

Digital images typically displayed on a graphics workstation monitor and stored on magnetic disk or magnetic tape

- Amount of data generated can quickly get quite large
- 1 entire micrograph scanned at **7  $\mu\text{m}$**  step size generates about  **$163 \times 10^6$  pixels** which translates into **326 Mb** of data (*i.e.* only 3 micrographs per Gbyte!!!)



## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 3. Boxing and Floating the Digitized Image

Entire digital image or selected (boxed) areas may be used for subsequent processing steps

To use only a portion of the scanned image:

- Area of interest is boxed (**windowed**) in manner similar to masking micrographs for OD or OF
- Thus, **areas outside** the biological specimen (*e.g.* carbon film or vitrified water or other “junk”), which mainly contribute noise to the image, are **selectively removed**

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 3. Boxing and Floating the Digitized Image

##### **Boxing:**

- Operation that **zeroes regions** in the digital image **outside** the area of interest (equivalent to “masking” in OD or OF experiments)
- Performed directly on digital image displayed on a computer graphics monitor

**Note:** with auto- or semi-automated boxing routines, human intervention is reduced or eliminated and so too the requirement for graphics display of the data

## III.D.3 Digital Fourier Analysis of Electron Micrographs

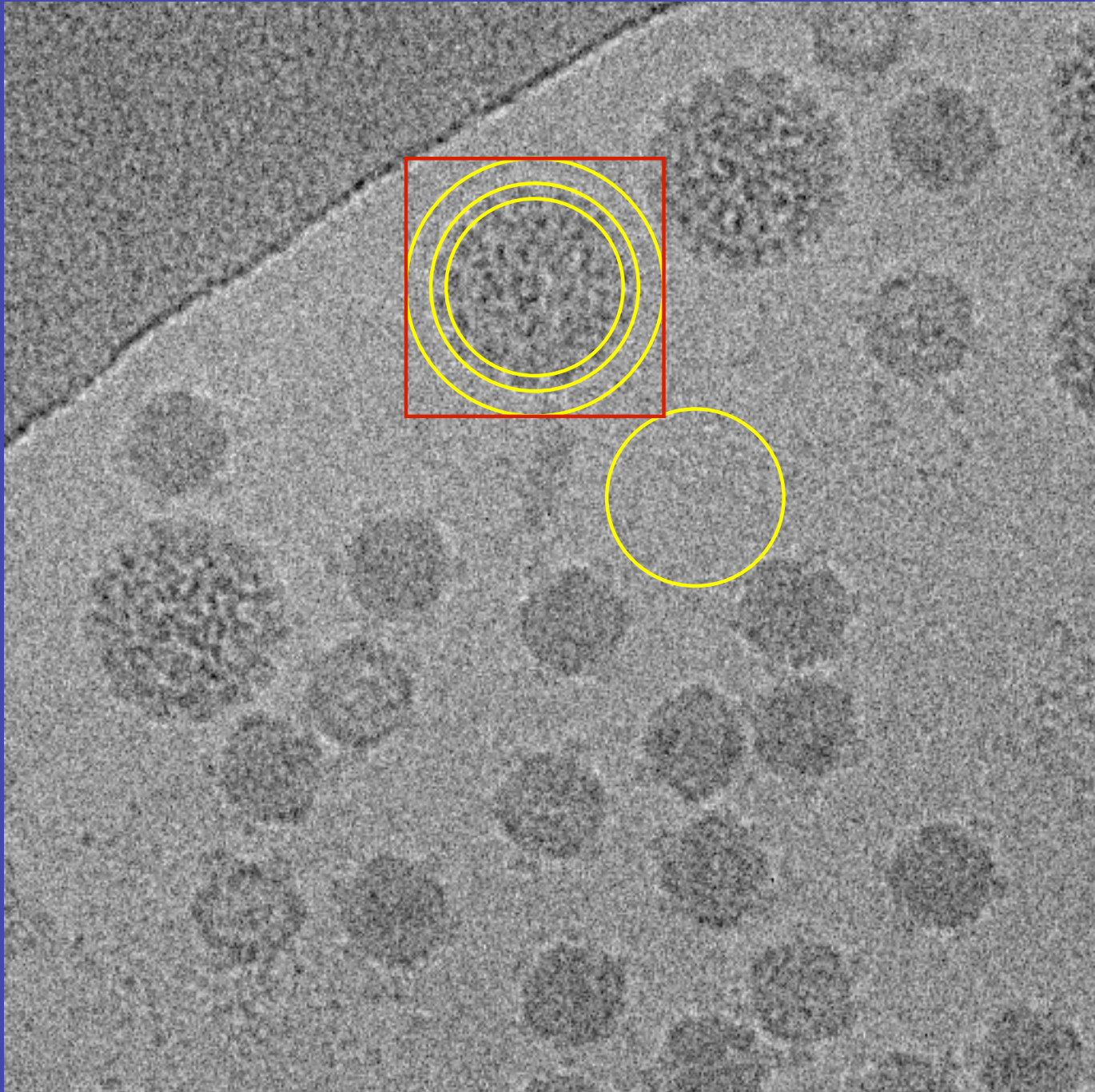
### III.D.3.b Digital Processing Steps

#### 3. Boxing and Floating the Digitized Image

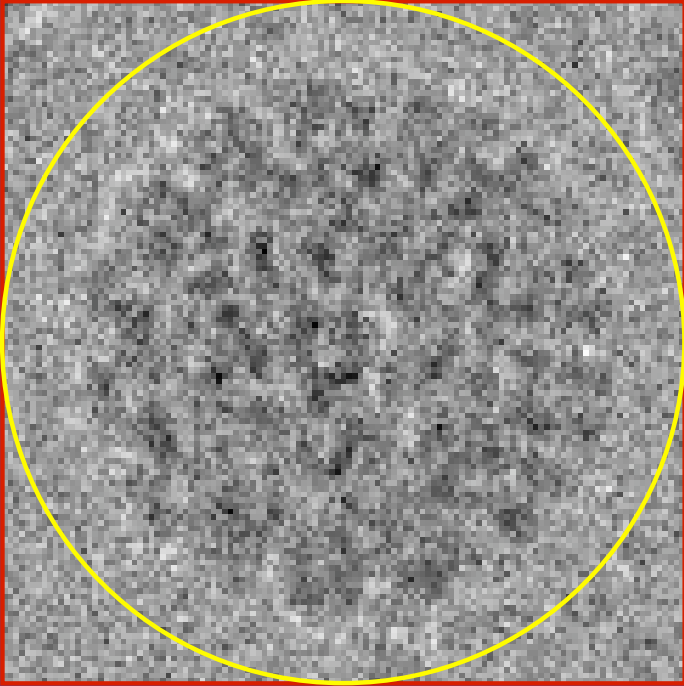
##### **Floating:**

- **Average** image intensity around **box perimeter** is **subtracted** from **all** image intensities **within** the masked area
- This suppresses strong diffraction "spikes" which arise from the high-contrast edges of the masked area

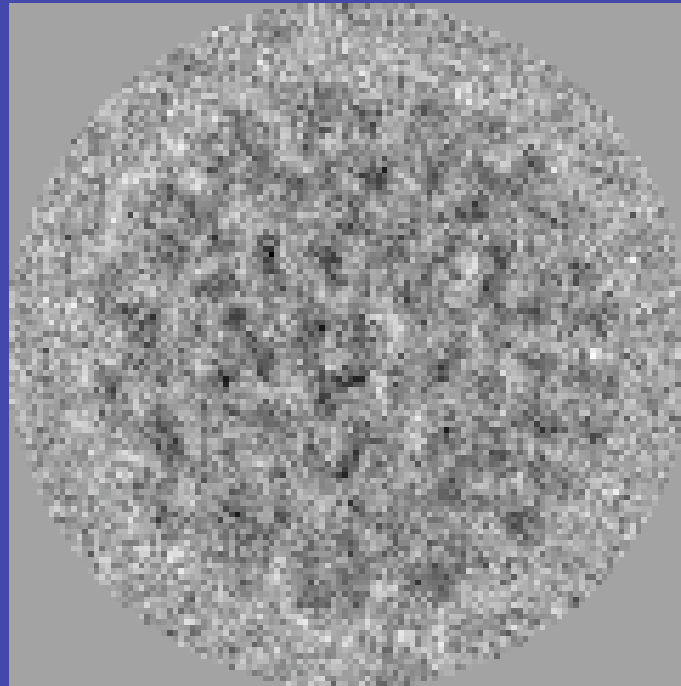
**Boxing:**



**Boxing:**

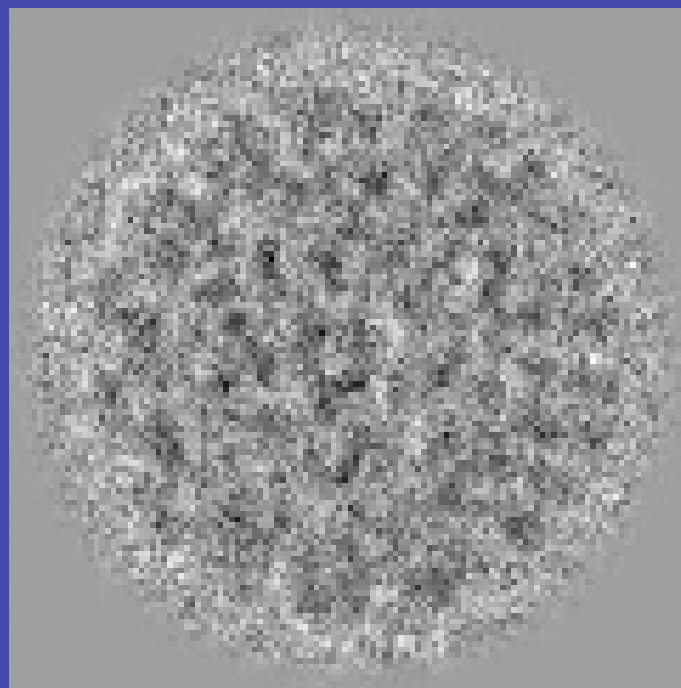


**Boxing:**



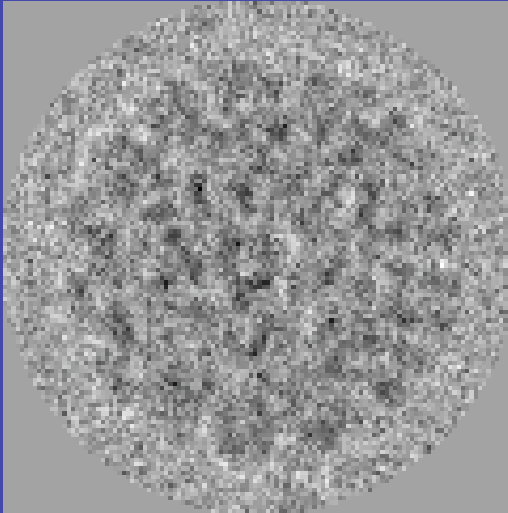
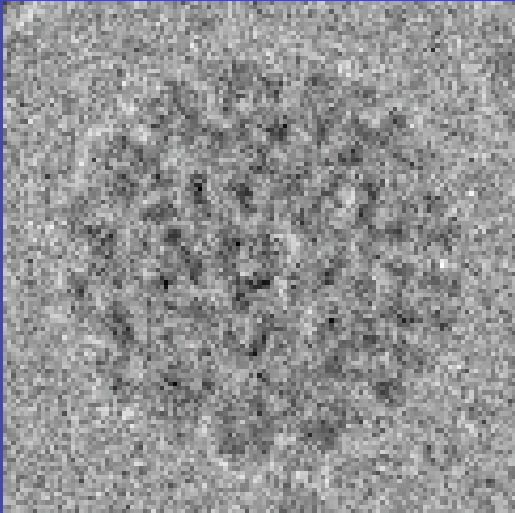
**Windowed**

**Boxing:**

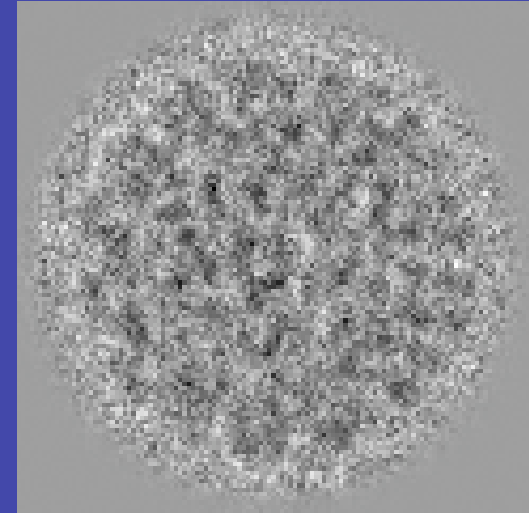


**Windowed  
and Apodized**

## Boxing:



Windowed



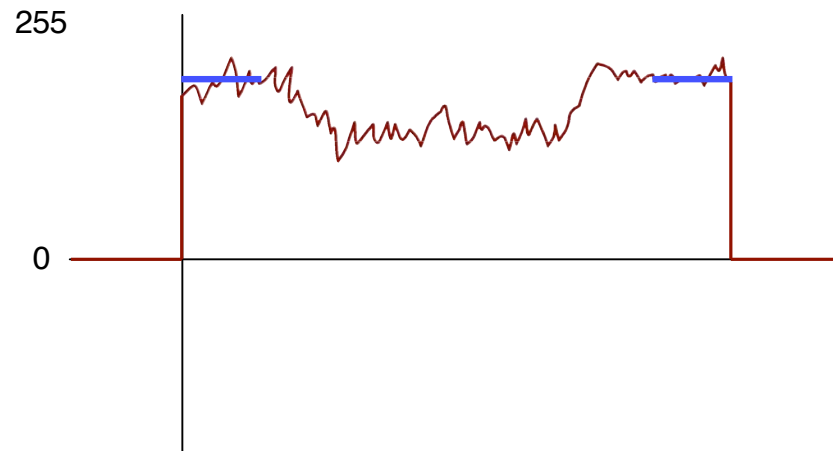
Windowed  
and Apodized



## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

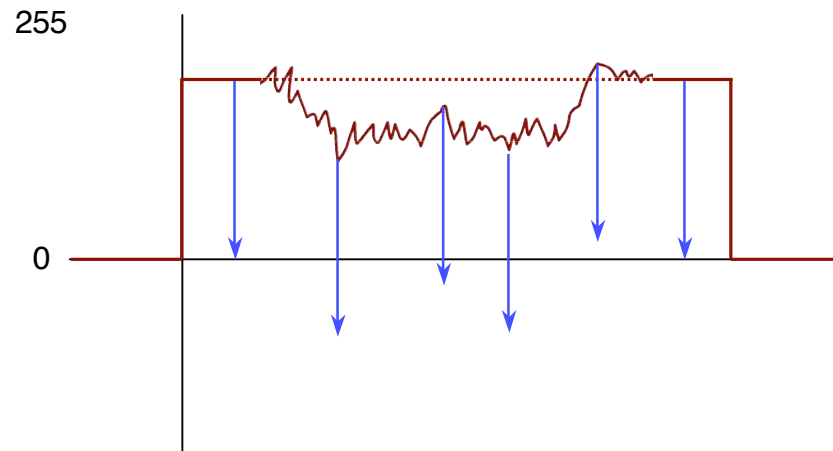
#### 3. Boxing and Floating the Digitized Image



## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

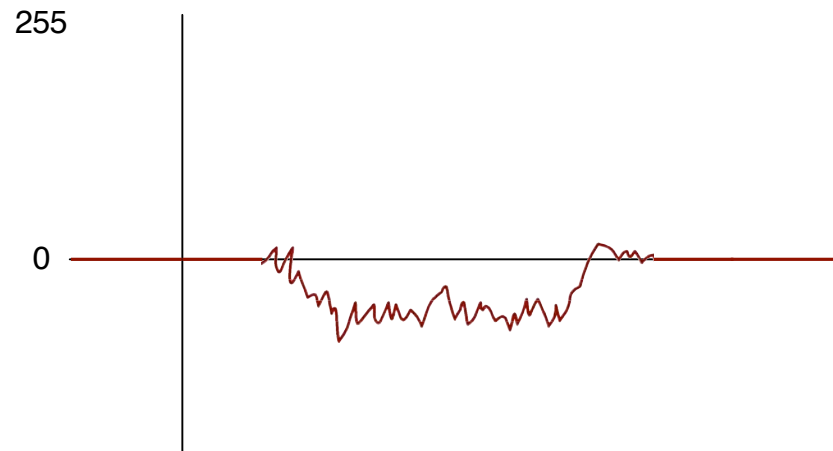
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## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

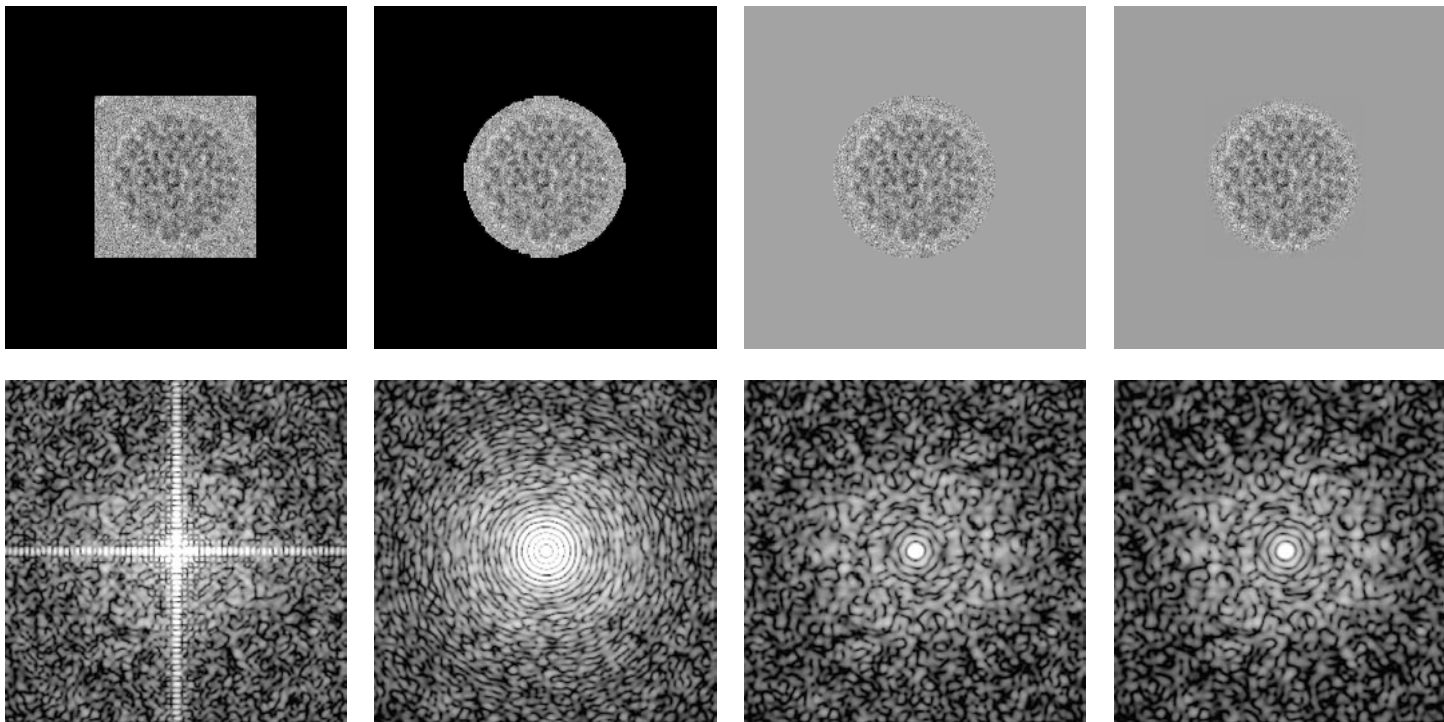
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## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

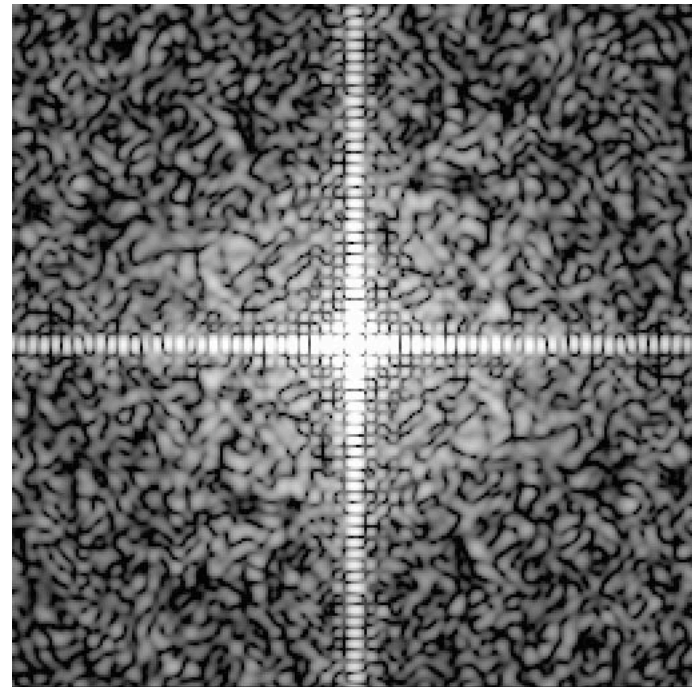
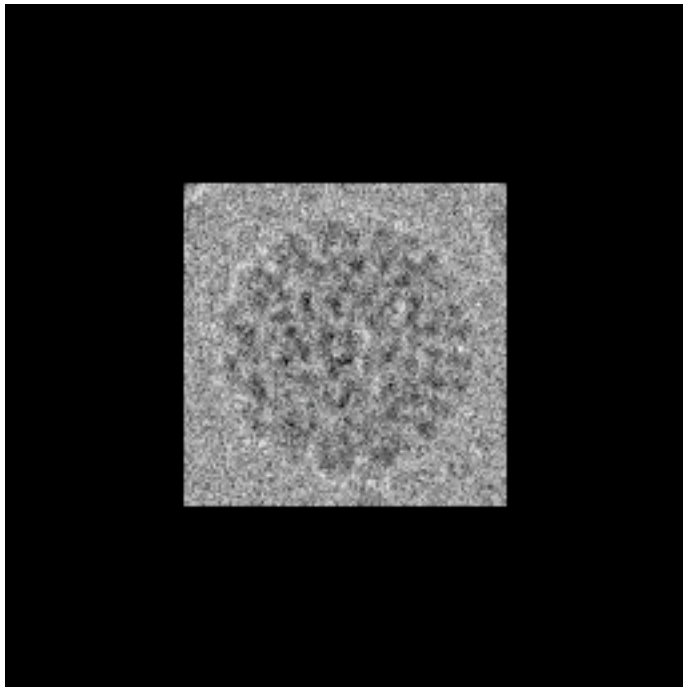
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### III.D.3.b Digital Processing Steps

#### 3. Boxing and Floating the Digitized Image

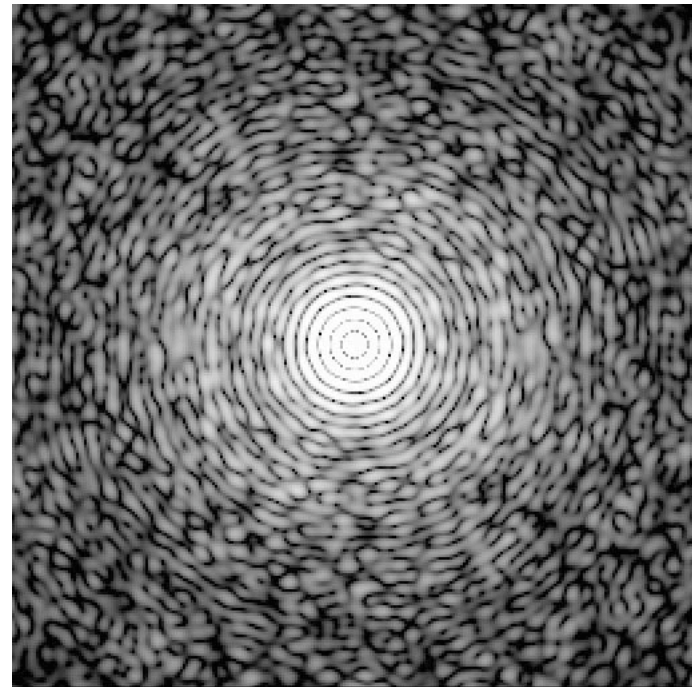
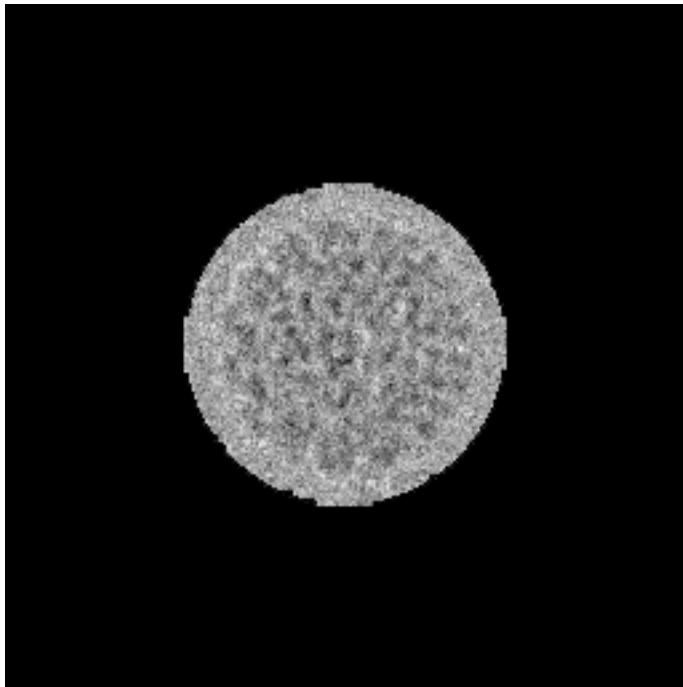


Square window; unfloated

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 3. Boxing and Floating the Digitized Image

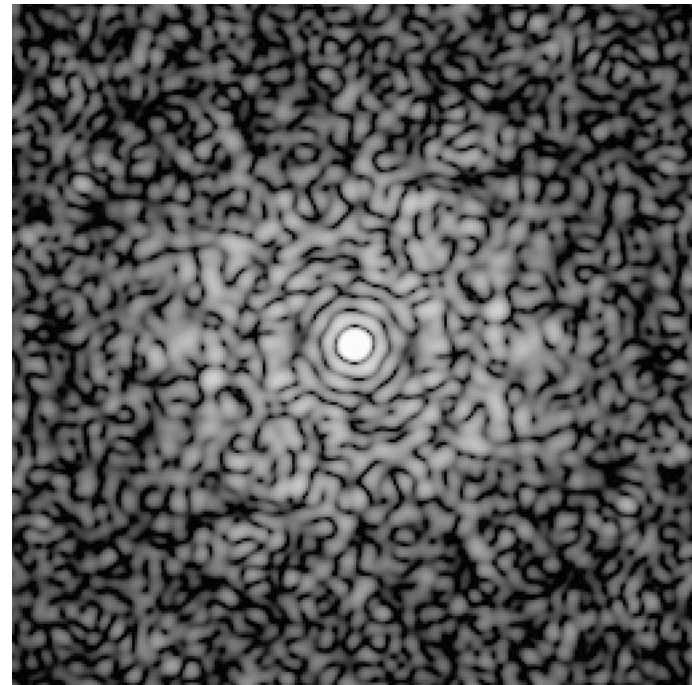
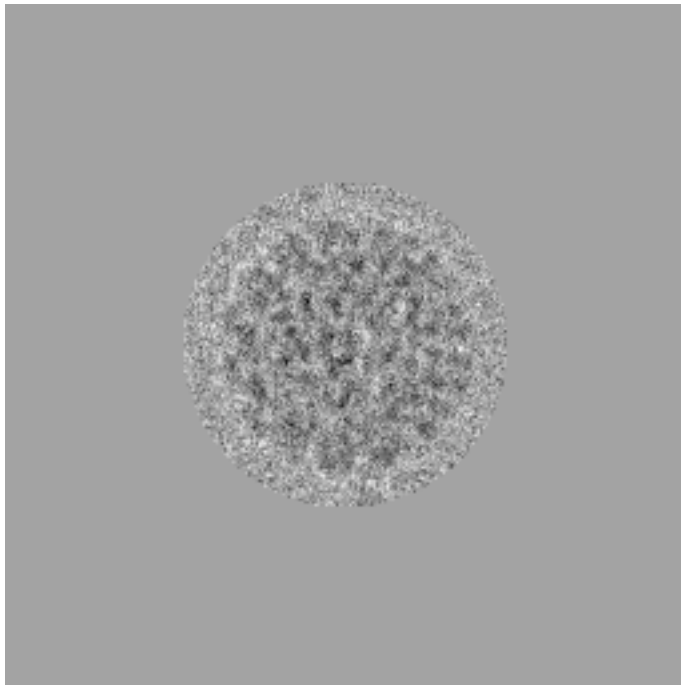


Circular window; unfloated

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 3. Boxing and Floating the Digitized Image

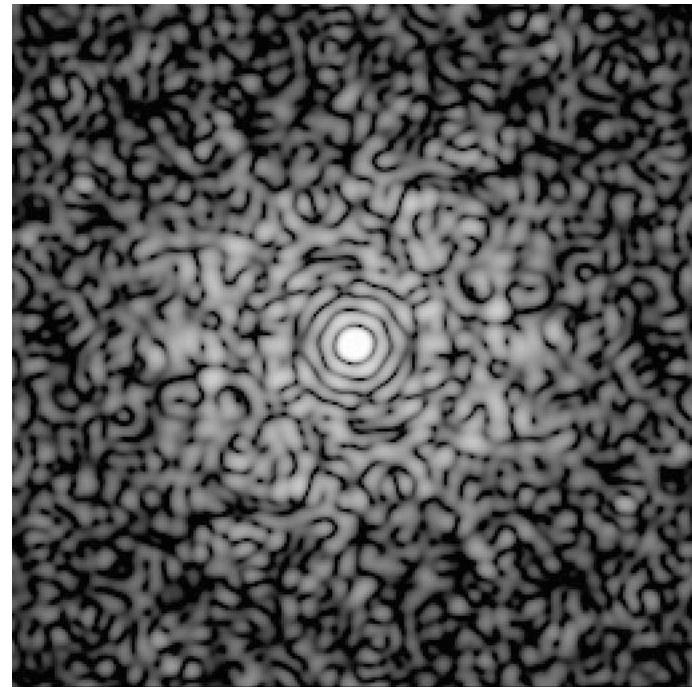
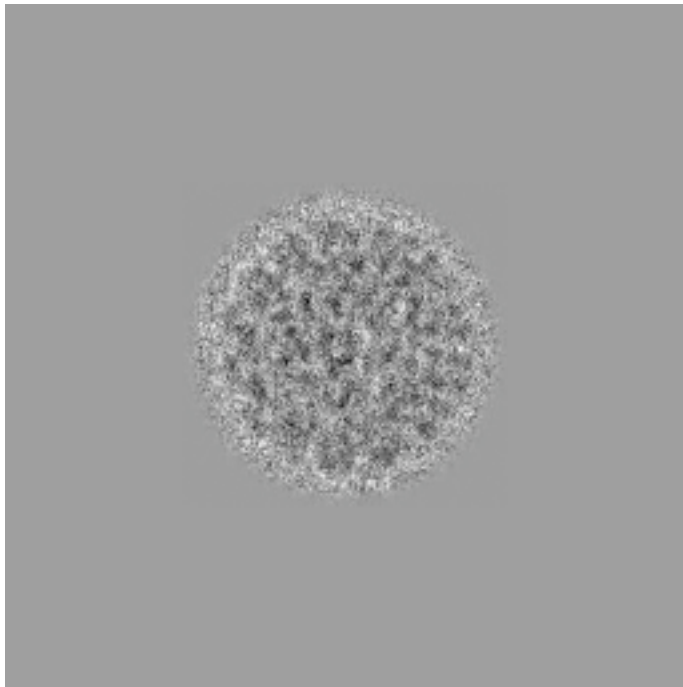


Circular window; floated

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 3. Boxing and Floating the Digitized Image



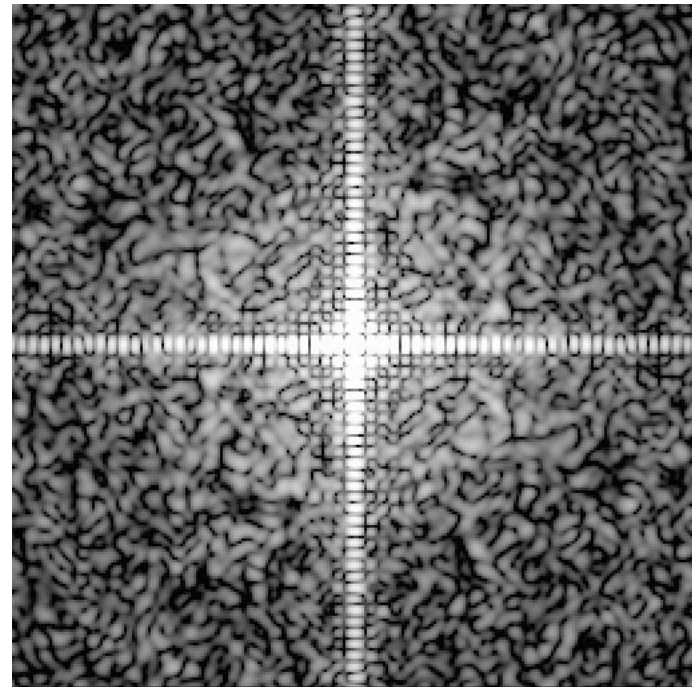
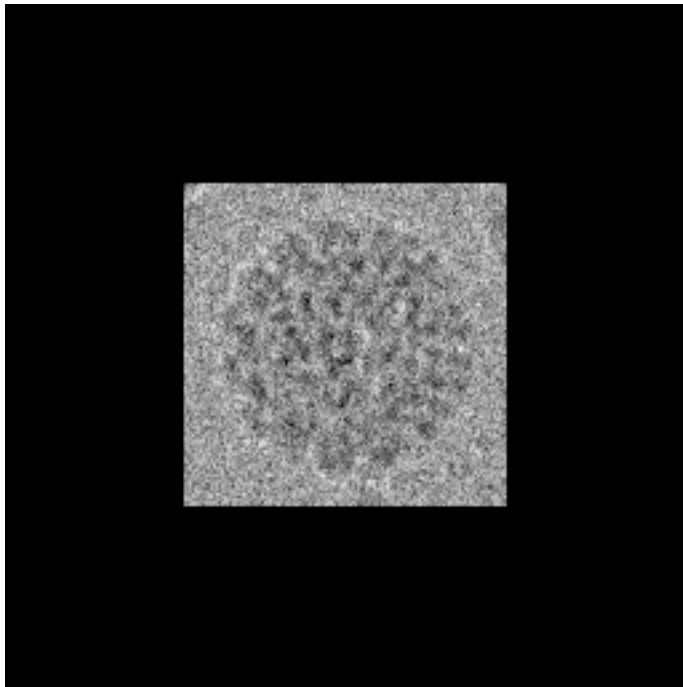
Circular window; apodized & floated



## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 3. Boxing and Floating the Digitized Image

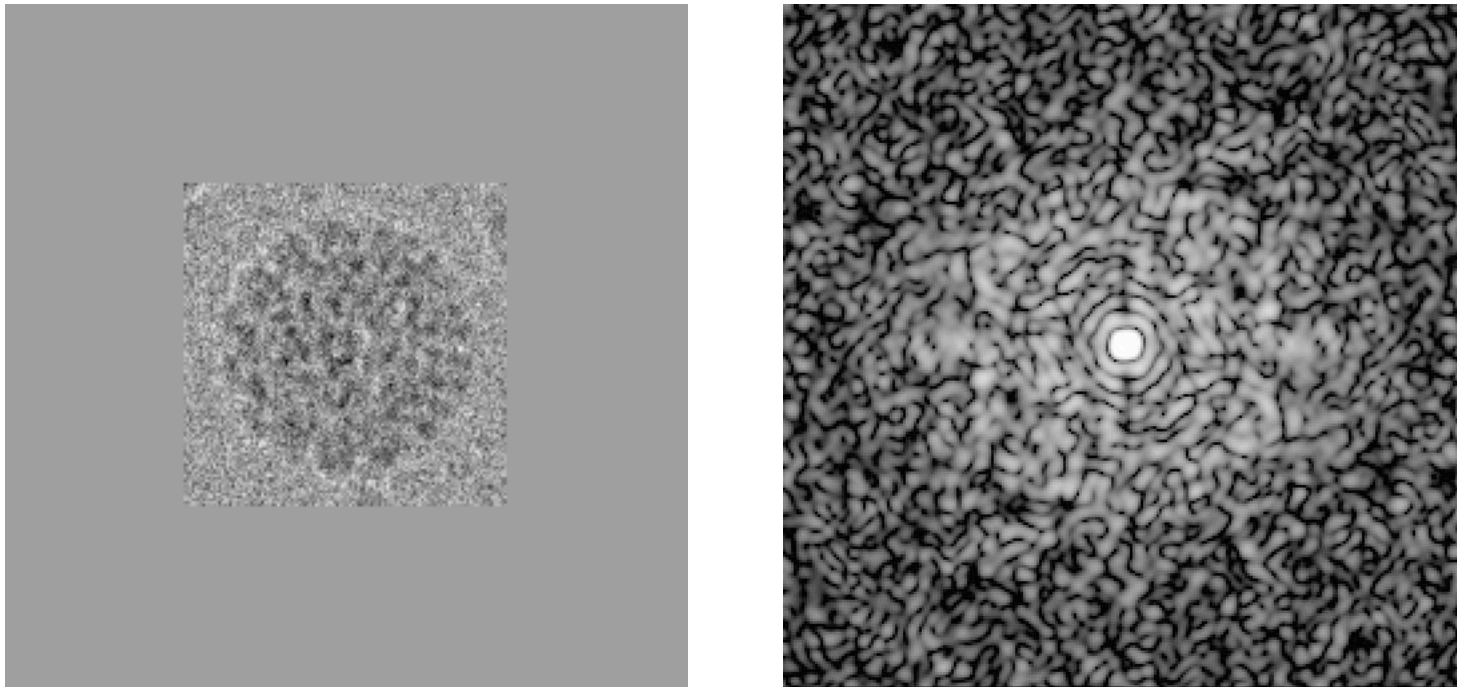


Square window; unfloated

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 3. Boxing and Floating the Digitized Image

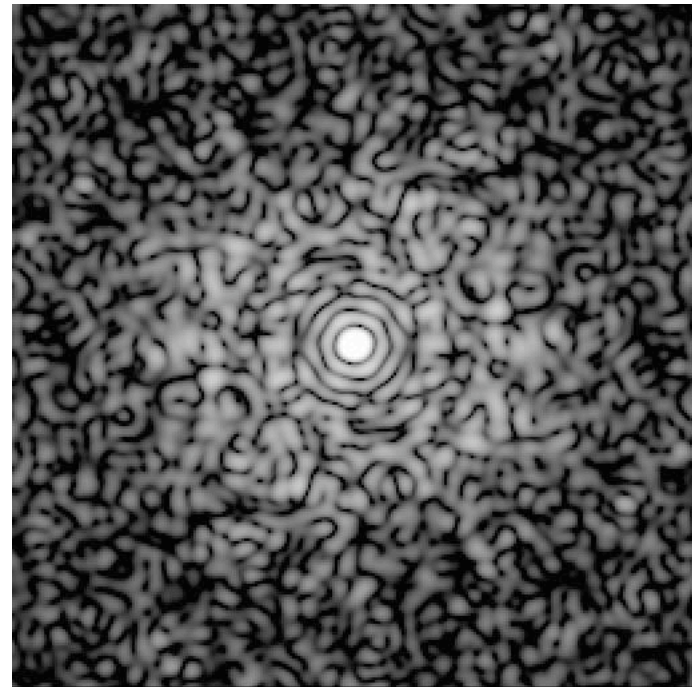
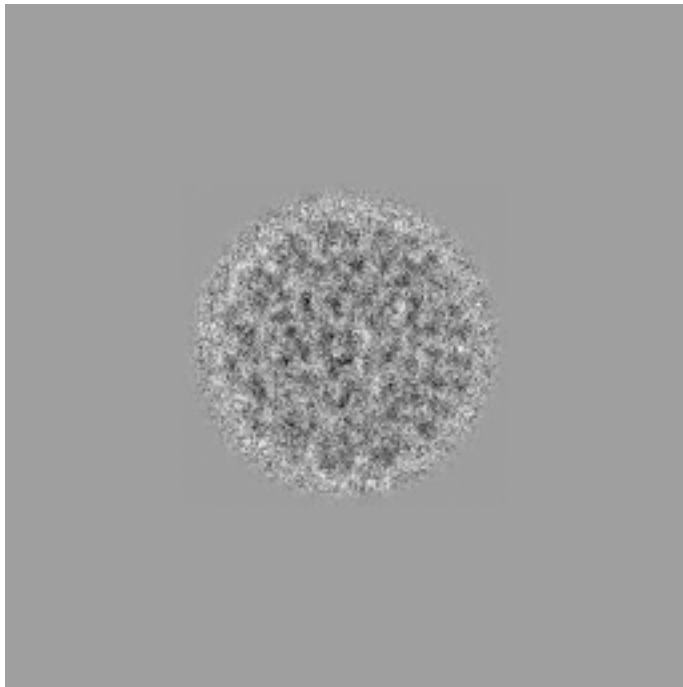


Square window; floated

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 3. Boxing and Floating the Digitized Image



Circular window; apodized & floated

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 4. Fourier Transformation

Fourier transform of numerical array computed by fast-Fourier (**FFT**) methods

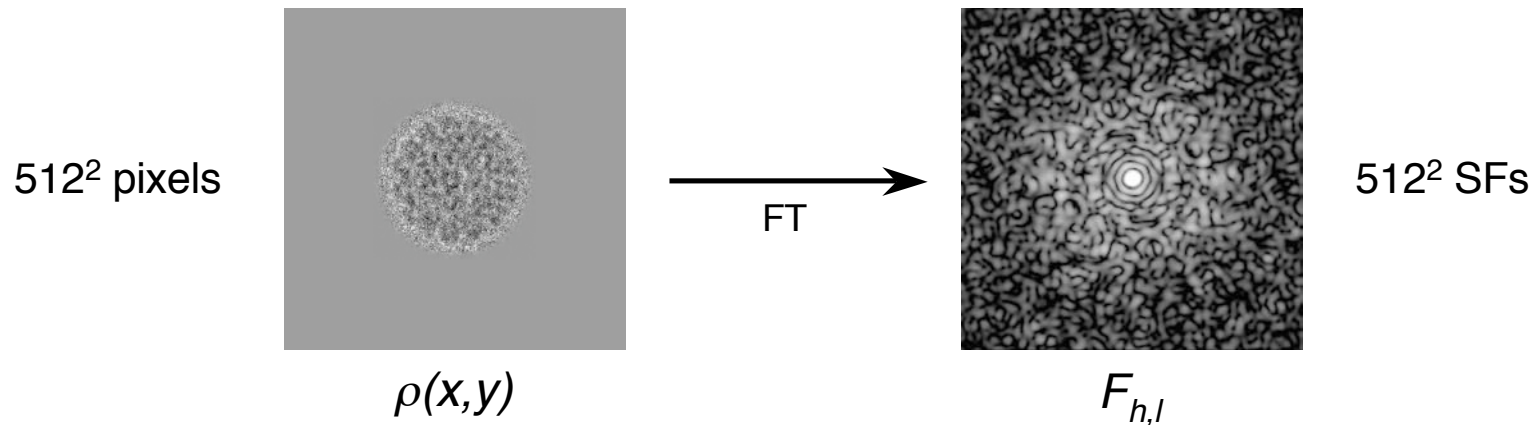
- Nothing magical or mystical to FFT routines
- Have been readily available for decades and are well tested

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 4. Fourier Transformation

Fourier transform of an  $n$  by  $m$  pixel image results in an  $n$  by  $m$  **complex** array of numbers (structure factors)



**Recall:**

$$F_{h,l} = \sum_y \sum_x \rho(x, y) e^{2\pi i(hx + ky)}$$

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 4. Fourier Transformation

Fourier transform of an  $n$  by  $m$  pixel image results in an  $n$  by  $m$  **complex** array of numbers (structure factors)

Each structure factor is stored in computer memory either as an **amplitude and phase** or as a **real** (A-part) **and imaginary** (B-part) part

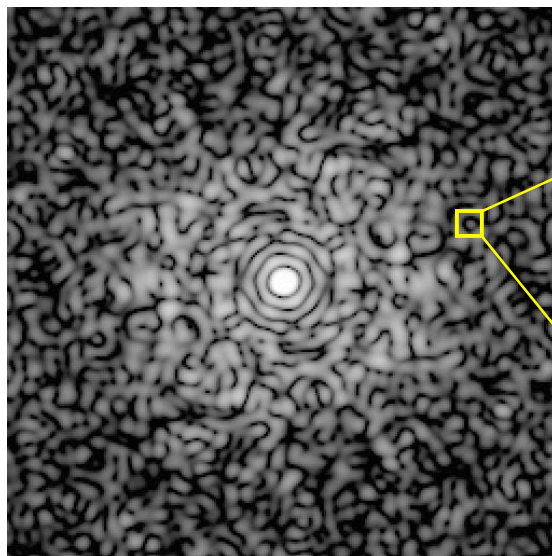
Transforms or diffraction patterns generally displayed on a graphics monitor (*e.g.* in RobEM)

# III.D.3 Digital Fourier Analysis of Electron Micrographs

## III.D.3.b Digital Processing Steps

### 4. Fourier Transformation

Fourier transform of an  $n$  by  $m$  pixel image results in an  $n$  by  $m$  **complex** array of numbers (structure factors)



AMPLITUDES										PHASES									
5	10	7	11	5	4	10	5	6		3	30	24	31	36	31	31	6	23	
6	6	21	9	3	8	1	13	9		33	18	10	9	31	7	9	9	10	
4	14	8	8	5	3	1	5	1		29	3	5	7	5	29	14	16	25	
9	16	15	7	8	11	7	4	5		28	1	4	1	13	14	2	21	20	
9	14	19	10	25	12	25	14	10		7	26	3	17	23	25	4	24	16	
10	10	13	15	40	27	15	18	7		1	20	30	15	23	31	9	25	33	
8	6	7	7	12	6	14	10	6		30	34	35	7	7	18	20	32	2	
9	5	6	6	21	16	8	3	6		10	2	31	20	22	24	34	25	16	
6	3	5	7	15	13	4	13	5		29	17	26	25	13	12	17	19	32	

$$|F_{h,k}|$$

$$\alpha_{h,k} / 10$$

$$F_{h,k}$$

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 5. Indexing of 2-D Lattices

Correct indexing of diffraction pattern **ESSENTIAL** for successful image reconstruction analysis

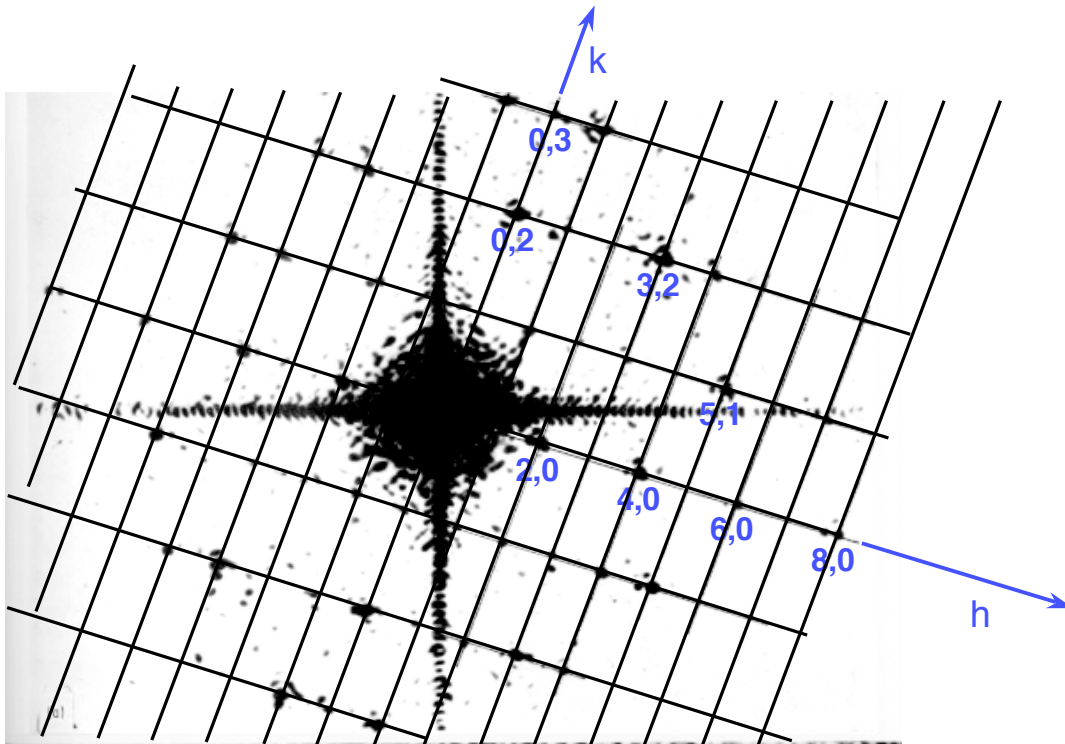
For a well-ordered biological specimen (*e.g.* 2D crystal):

- Diffraction pattern is a series of discrete, sharp spots (Bragg reflections) on a reciprocal lattice
- Such patterns usually fairly easy to index (*i.e.* define reciprocal lattice parameters and assign a Miller index to each spot)

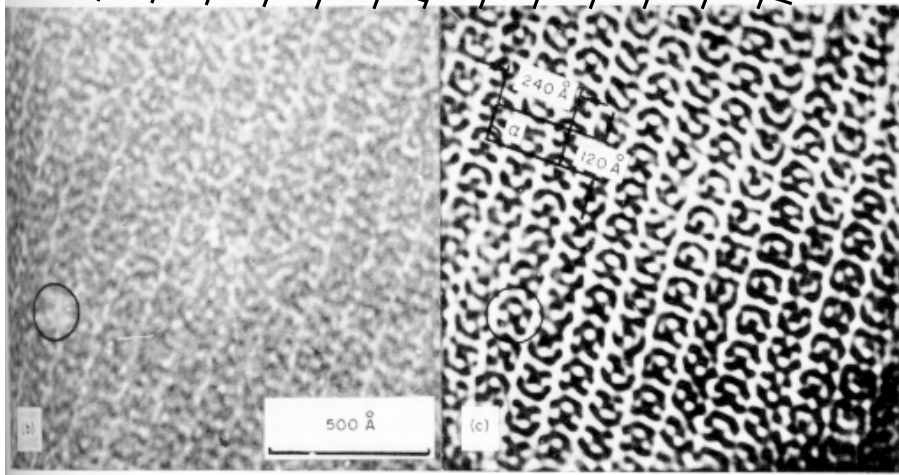
**Example:** Phosphorylase b crystal



### III.D.2.a Indexing the Optical Diffraction Pattern



OD and Filtration of  
Negatively-stained  
Phosphorylase b Crystal



Unfiltered

Filtered

From Kiselev et al., (1971) Plate III

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 5. Indexing of 2-D Lattices

For multilayered or two-sided structures (*e.g.* biological aggregates with helical symmetry):

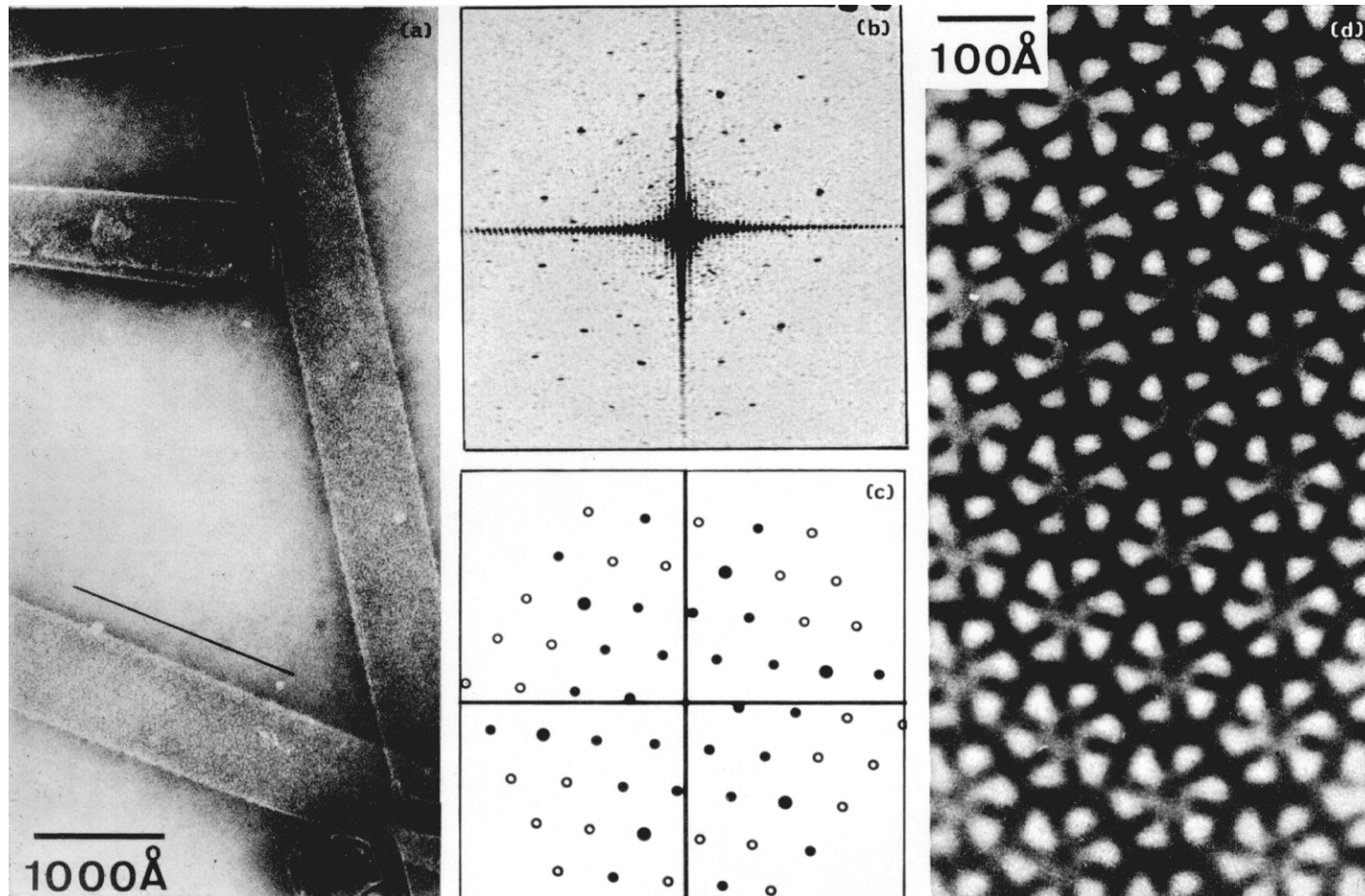
- Indexing can be quite tricky

**Example:** T4 Polyhead

### III.D.3 Digital Fourier Analysis of Electron Micrographs

#### III.D.3.b Digital Processing Steps

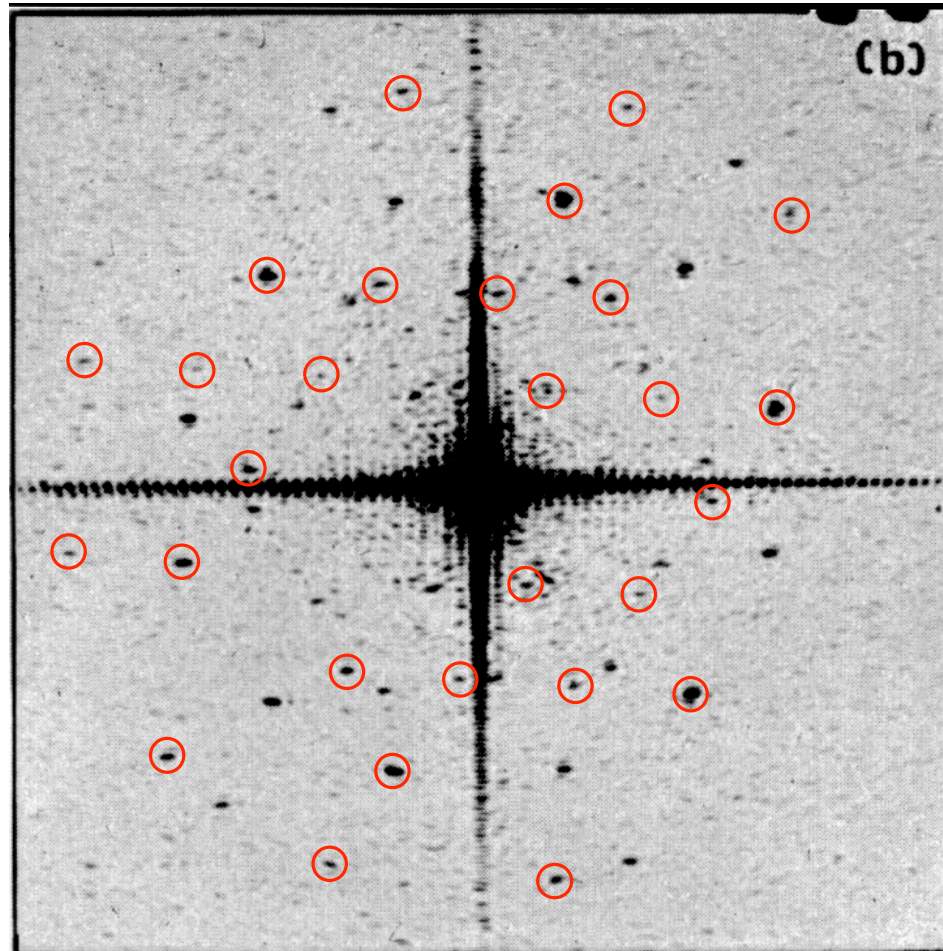
#### 5. Indexing of 2-D Lattices



## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

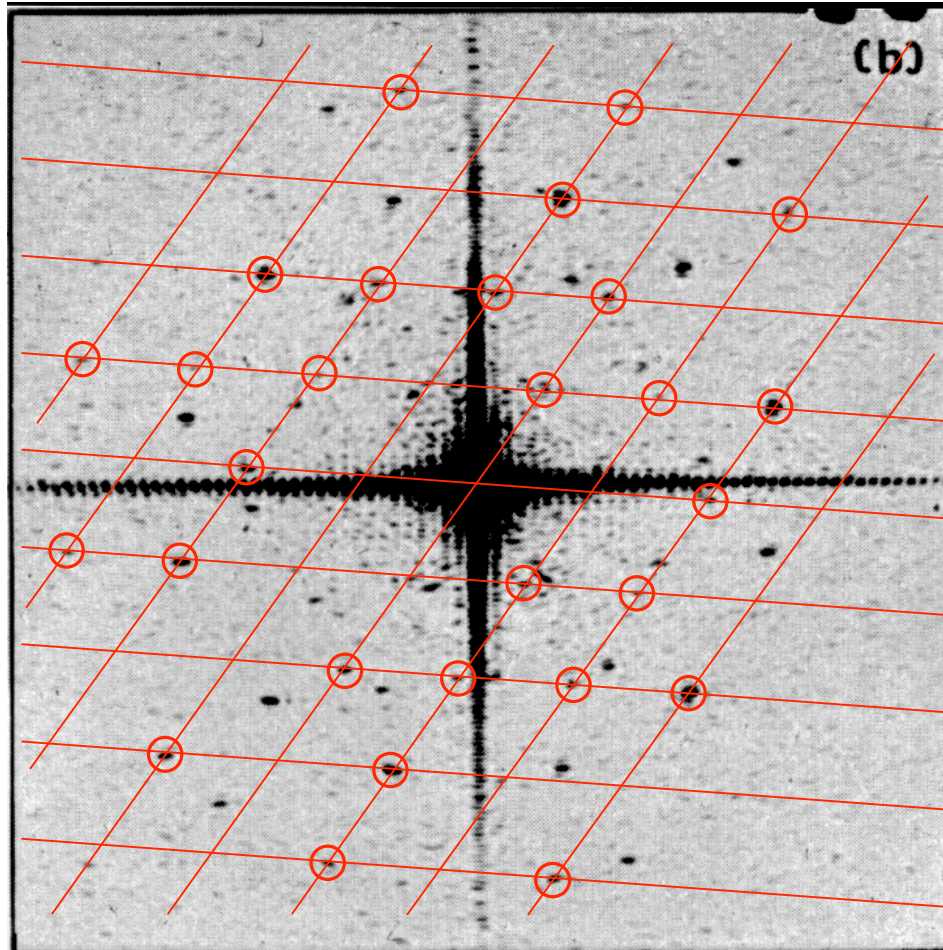
#### 5. Indexing of 2-D Lattices



## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

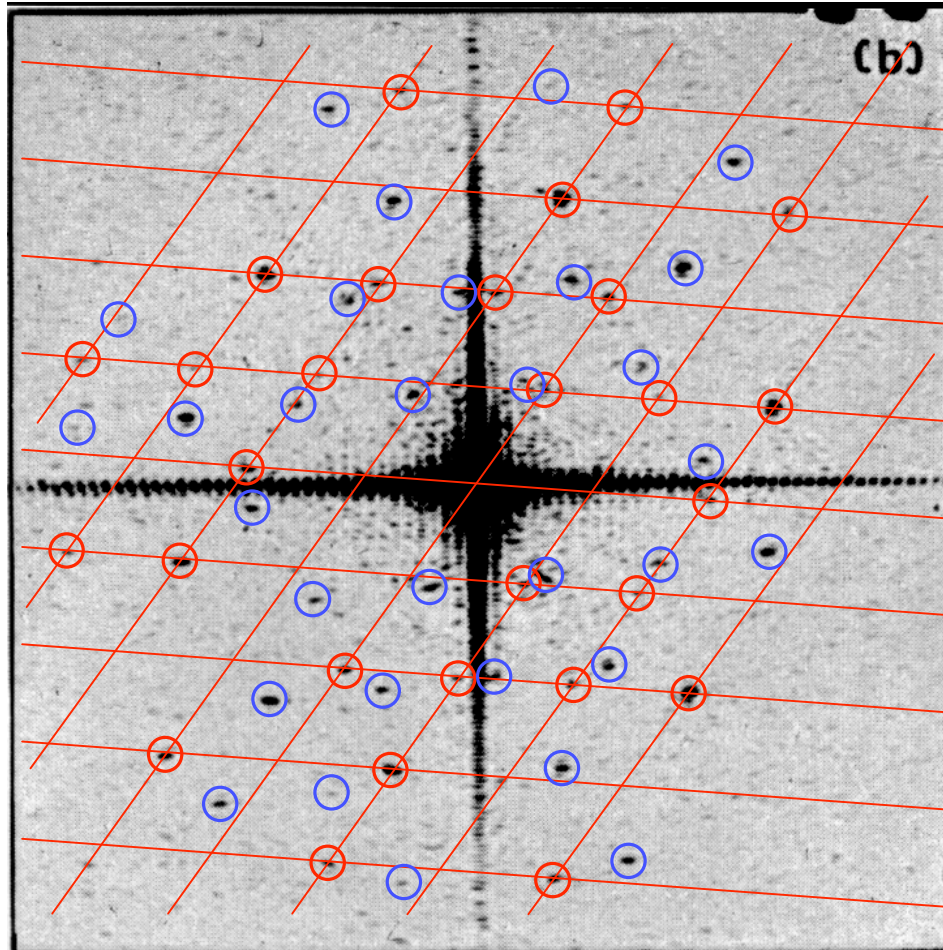
#### 5. Indexing of 2-D Lattices



## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

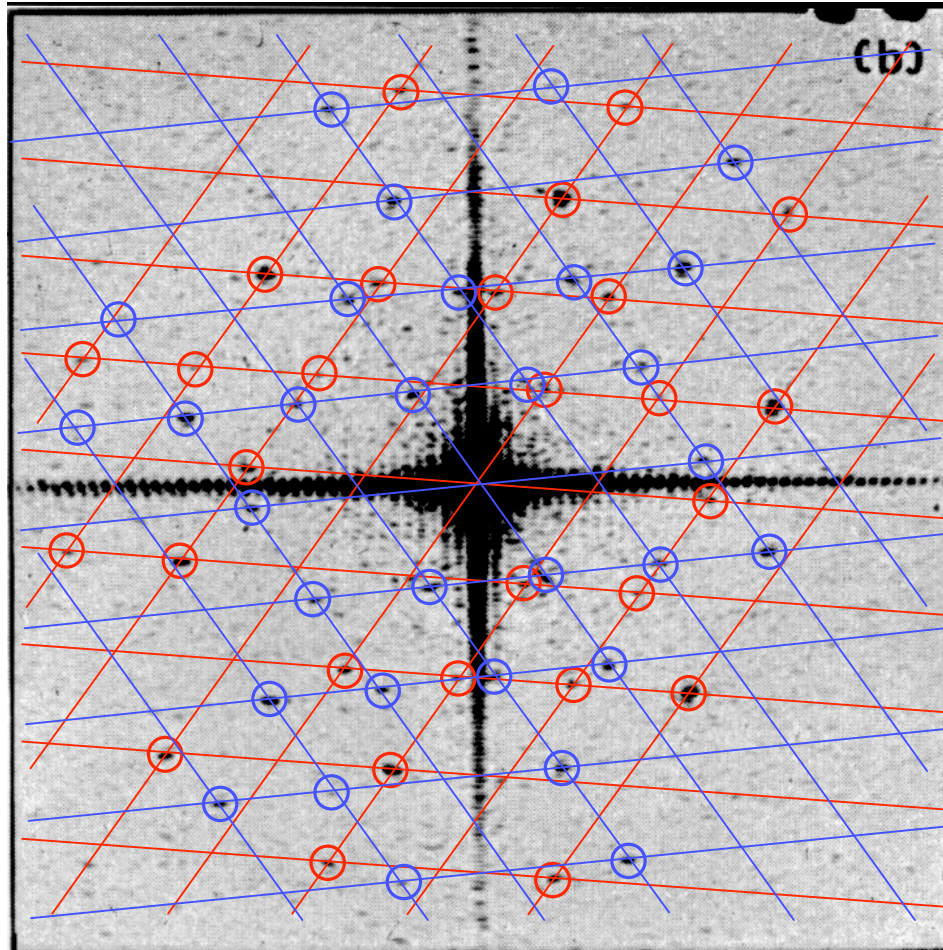
#### 5. Indexing of 2-D Lattices



## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

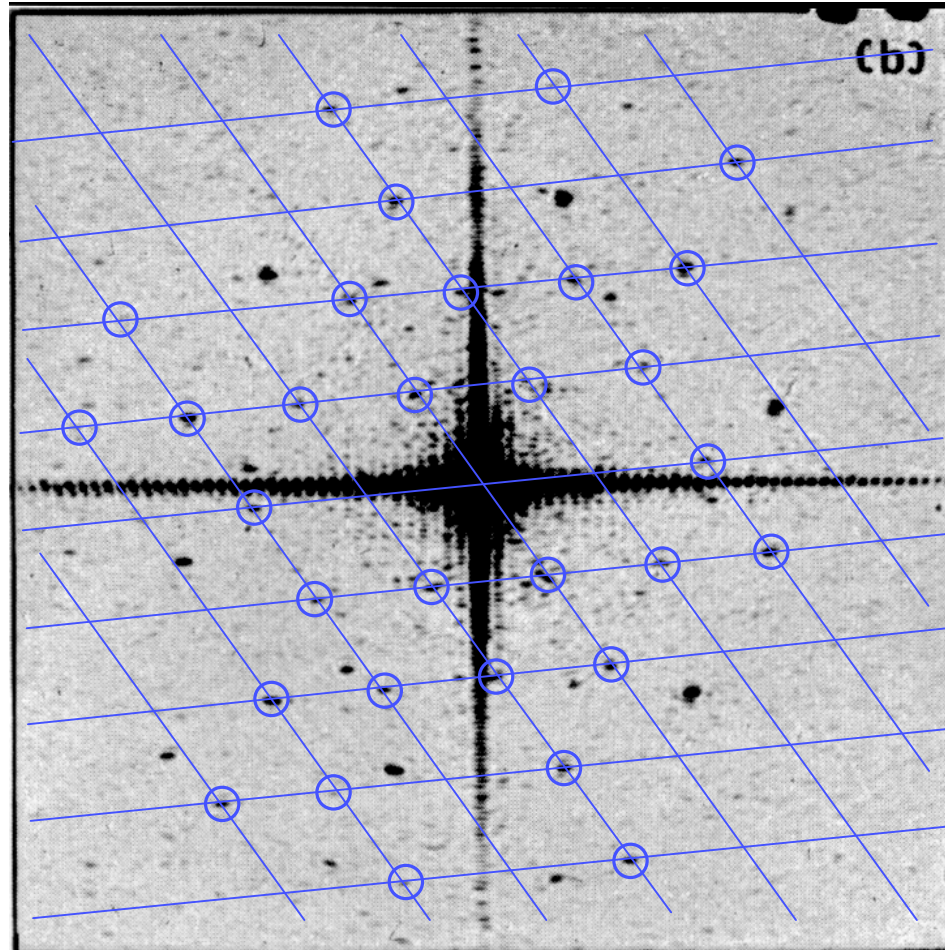
#### 5. Indexing of 2-D Lattices



## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 5. Indexing of 2-D Lattices





### III.D.3 Digital Fourier Analysis of Electron Micrographs

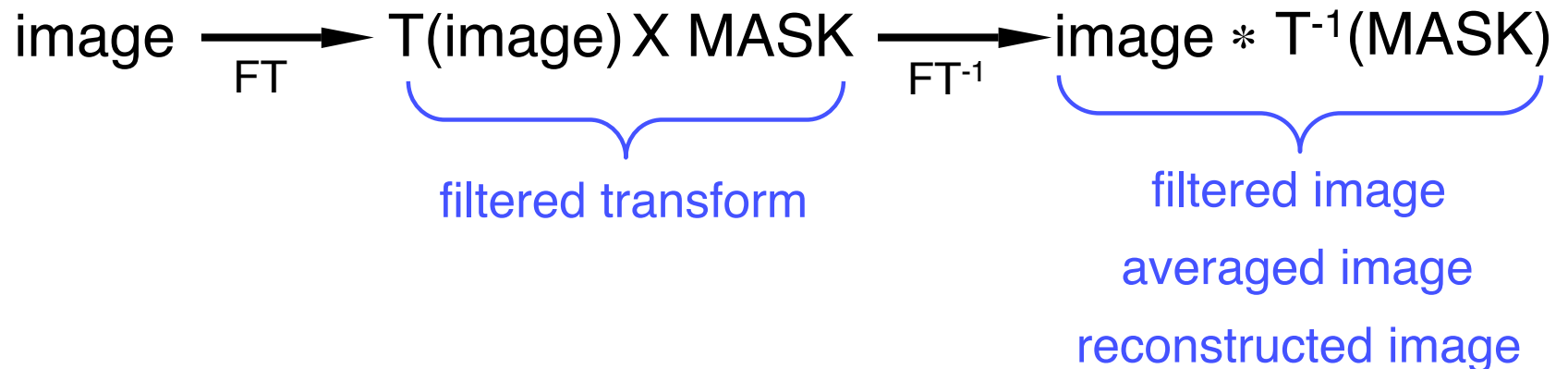
#### III.D.3.b Digital Processing Steps

#### 6. 2-D Filtering / 3-D Reconstruction

Amplitudes in computed FT **zeroed** everywhere except at or near reciprocal lattice points

An averaged image is reconstructed by back-transforming the modified ("filtered") diffraction pattern

#### Recall:



## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 6. 2-D Filtering / 3-D Reconstruction

#### “Pseudo-Optical” Filtering:

Here, "points" actually refers to **finite regions** (holes in the "filter mask") that surround the points of an **ideal** reciprocal lattice

Data inside the mask holes are left as is (*i.e.* multiplied by 1) or may be weighted according to the distance of each transform data value from the ideal lattice

*Will demo this next time in RobEM*

### III.D.3.b Digital Processing Steps

#### 6. 2-D Filtering / 3-D Reconstruction

Amplitudes ( $|F_{h,k}|$ )

Phases ( $\alpha_{h,k} / 10$ )

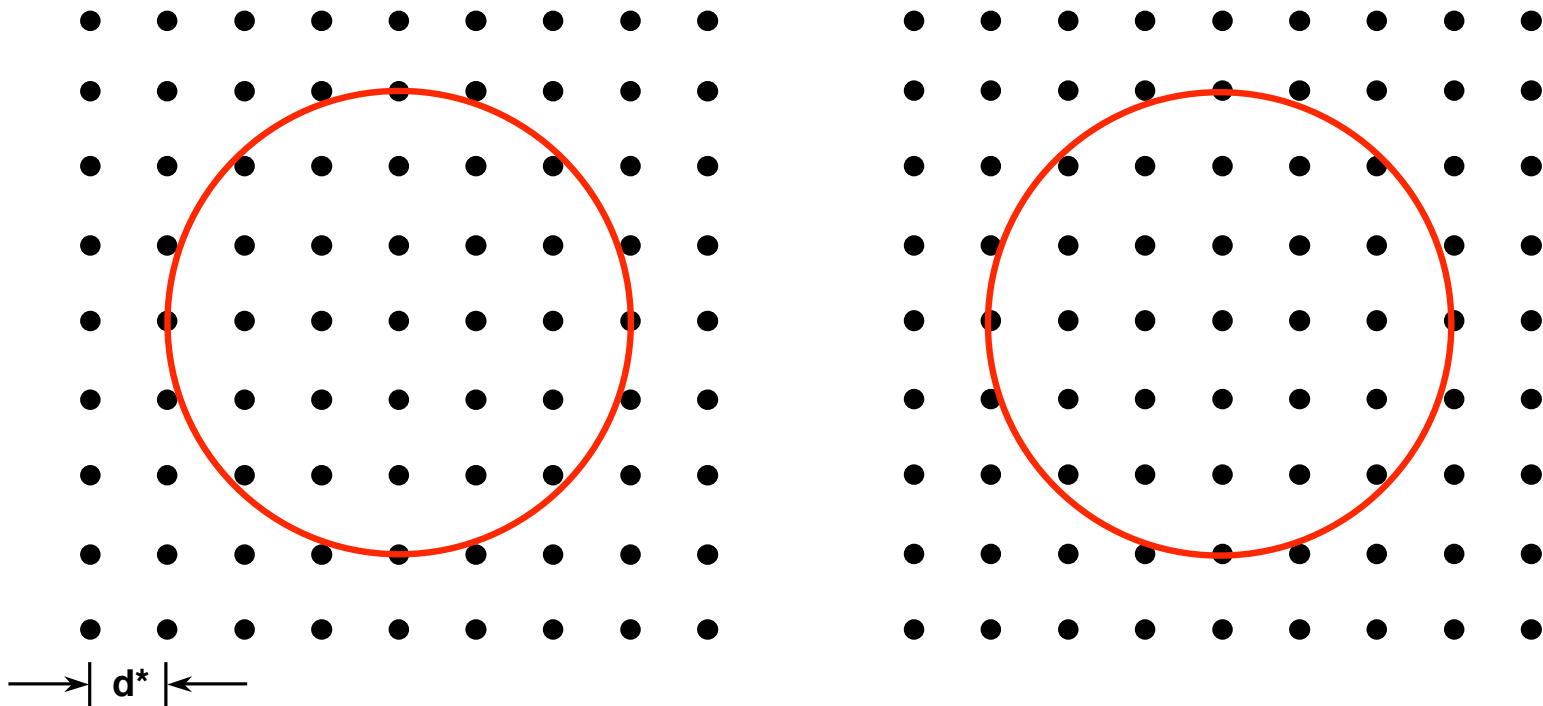
5	10	7	11	5	4	10	5	6	3	30	24	31	36	31	31	6	23
6	6	21	9	3	8	1	13	9	33	18	10	9	31	7	9	9	10
4	14	8	8	5	3	1	5	1	29	3	5	7	5	29	14	16	25
9	16	15	7	8	11	7	4	5	28	1	4	1	13	14	2	21	20
9	14	19	10	85	18	25	14	10	7	26	3	17	23	25	4	24	16
10	10	13	15	46	27	15	18	7	1	20	30	15	23	31	9	25	33
8	6	7	7	12	6	14	10	6	30	34	35	7	7	18	20	32	2
9	5	6	6	21	16	8	3	6	10	2	31	20	22	24	34	25	16
6	3	5	7	15	13	4	13	5	29	17	26	25	13	12	17	19	32

### III.D.3.b Digital Processing Steps

#### 6. 2-D Filtering / 3-D Reconstruction

Amplitudes ( $|F_{h,k}|$ )

Phases ( $\alpha_{h,k} / 10$ )



$$D_{\text{HOLE}} = 6d^*$$

### III.D.3.b Digital Processing Steps

#### 6. 2-D Filtering / 3-D Reconstruction

Amplitudes ( $|F_{h,k}|$ )

Phases ( $\alpha_{h,k} / 10$ )

5	10	7	11	5	4	10	5	6
6	6	21	9	3	8	1	13	9
4	14	8	8	5	3	1	5	1
9	16	15	7	8	11	7	4	5
9	14	19	10	85	18	25	14	10
10	10	13	15	46	27	15	18	7
8	6	7	7	12	6	14	10	6
9	5	6	6	21	16	8	3	6
6	3	5	7	15	13	4	13	5

3	30	24	31	36	31	31	6	23
33	18	10	9	31	7	9	9	10
29	3	5	7	5	29	14	16	25
28	1	4	1	13	14	2	21	20
7	26	3	17	23	25	4	24	16
1	20	30	15	23	31	9	25	33
30	34	35	7	7	18	20	32	2
10	2	31	20	22	24	34	25	16
29	17	26	25	13	12	17	19	32

→ | d\* | ←

$$D_{\text{HOLE}} = 6d^*$$

### III.D.3.b Digital Processing Steps

#### 6. 2-D Filtering / 3-D Reconstruction

Amplitudes ( $|F_{h,k}|$ )

0	0	0	0	0	0	0	0	0
0	0	0	0	3	0	0	0	0
0	0	8	8	5	3	1	0	0
0	0	15	7	8	11	7	0	0
0	14	19	10	85	18	25	14	0
0	0	13	15	46	27	15	0	0
0	0	7	7	12	6	14	0	0
0	0	0	0	21	0	0	0	0
0	0	0	0	0	0	0	0	0

Phases ( $\alpha_{h,k} / 10$ )

3	30	24	31	36	31	31	6	23
33	18	10	9	31	7	9	9	10
29	3	5	7	5	29	14	16	25
28	1	4	1	13	14	2	21	20
7	26	3	17	23	25	4	24	16
1	20	30	15	23	31	9	25	33
30	34	35	7	7	18	20	32	2
10	2	31	20	22	24	34	25	16
29	17	26	25	13	12	17	19	32

→ |  $d^*$  | ←

$$D_{\text{HOLE}} = 6d^*$$

### III.D.3.b Digital Processing Steps

#### 6. 2-D Filtering / 3-D Reconstruction

Amplitudes ( $|F_{h,k}|$ )

Phases ( $\alpha_{h,k} / 10$ )

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	5	0	0	0	0
0	0	0	7	8	11	0	0	0
0	0	19	10	85	18	25	0	0
0	0	0	15	46	27	0	0	0
0	0	0	0	12	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

3	30	24	31	36	31	31	6	23
33	18	10	9	31	7	9	9	10
29	3	5	7	5	29	14	16	25
28	1	4	1	13	14	2	21	20
7	26	3	17	23	25	4	24	16
1	20	30	15	23	31	9	25	33
30	34	35	7	7	18	20	32	2
10	2	31	20	22	24	34	25	16
29	17	26	25	13	12	17	19	32

→ | d\* | ←

$$D_{\text{HOLE}} = 4d^*$$

### III.D.3.b Digital Processing Steps

## 6. 2-D Filtering / 3-D Reconstruction

Amplitudes ( $ F_{h,k} $ )									Phases ( $\alpha_{h,k} / 10$ )								
0	0	0	0	0	0	0	0	0	3	30	24	31	36	31	31	6	23
0	0	0	0	0	0	0	0	0	33	18	10	9	31	7	9	9	10
0	0	0	0	0	0	0	0	0	29	3	5	7	5	29	14	16	25
0	0	0	0	8	0	0	0	0	28	1	4	1	13	14	2	21	20
0	0	0	10	85	18	0	0	0	7	26	3	17	23	25	4	24	16
0	0	0	0	46	0	0	0	0	1	20	30	15	23	31	9	25	33
0	0	0	0	0	0	0	0	0	30	34	35	7	7	18	20	32	2
0	0	0	0	0	0	0	0	0	10	2	31	20	22	24	34	25	16
0	0	0	0	0	0	0	0	0	29	17	26	25	13	12	17	19	32



$$D_{\text{HOLE}} = 2d^*$$



### III.D.3.b Digital Processing Steps

#### 6. 2-D Filtering / 3-D Reconstruction

Amplitudes ( $|F_{h,k}|$ )

Phases ( $\alpha_{h,k} / 10$ )

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	85	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

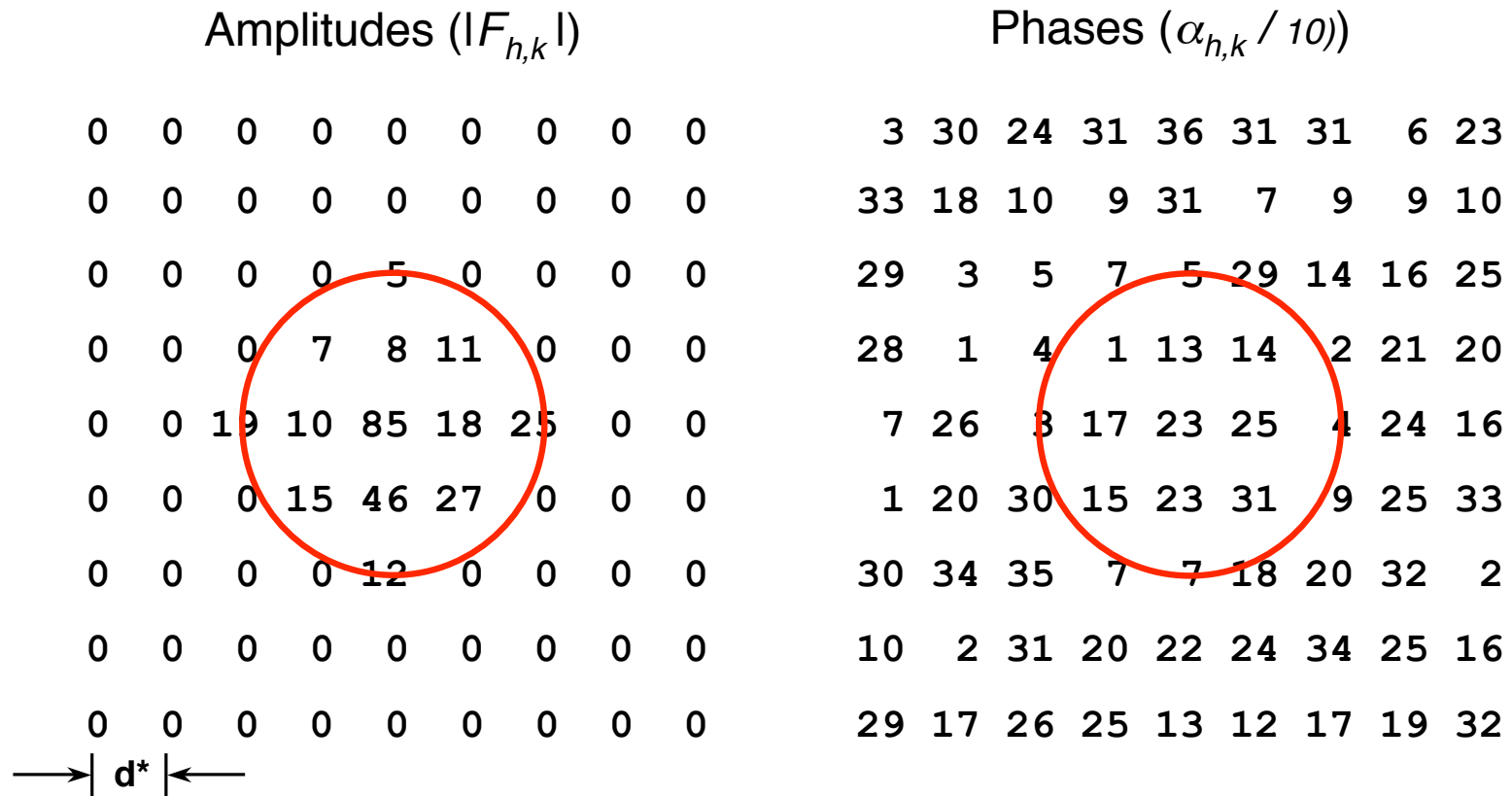
3	30	24	31	36	31	31	6	23
33	18	10	9	31	7	9	9	10
29	3	5	7	5	29	14	16	25
28	1	4	1	13	14	2	21	20
7	26	3	17	23	25	4	24	16
1	20	30	15	23	31	9	25	33
30	34	35	7	7	18	20	32	2
10	2	31	20	22	24	34	25	16
29	17	26	25	13	12	17	19	32

→ | d\* | ←

$$D_{\text{HOLE}} = d^*$$

### III.D.3.b Digital Processing Steps

## 6. 2-D Filtering / 3-D Reconstruction

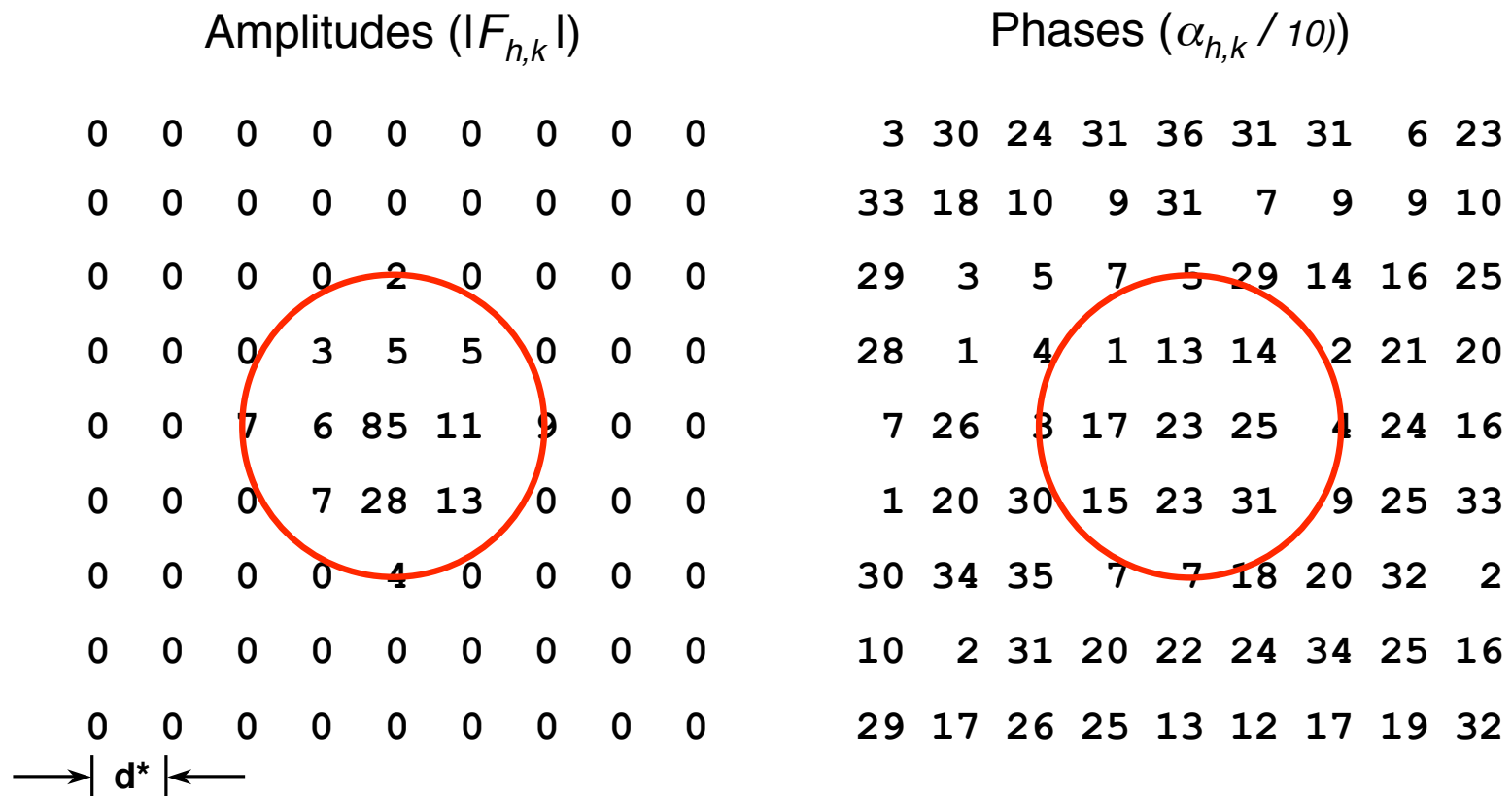


$$D_{\text{HOLE}} = 4d^*$$

“Hard” edge

### III.D.3.b Digital Processing Steps

## 6. 2-D Filtering / 3-D Reconstruction



$$D_{\text{HOLE}} = 4d^*$$

“Soft” edge

### III.D.3.b Digital Processing Steps

#### 6. 2-D Filtering / 3-D Reconstruction

Amplitudes ( $|F_{h,k}|$ )

5	10	7	11	5	4	10	5	6
6	6	21	9	3	8	1	13	9
4	14	8	8	5	3	1	5	1
9	16	15	7	8	11	7	4	5
9	14	19	10	<del>8</del>	18	25	14	10
10	10	13	15	46	27	15	18	7
8	6	7	7	12	6	14	10	6
9	5	6	6	21	16	8	3	6
6	3	5	7	15	13	4	13	5

Phases ( $\alpha_{h,k} / 10$ )

3	30	24	31	36	31	31	6	23
33	18	10	9	31	7	9	9	10
29	3	5	7	5	29	14	16	25
28	1	4	1	13	14	2	21	20
7	26	3	17	<del>23</del>	25	4	24	16
1	20	30	15	23	31	9	25	33
30	34	35	7	7	18	20	32	2
10	2	31	20	22	24	34	25	16
29	17	26	25	13	12	17	19	32

### III.D.3.b Digital Processing Steps

#### 6. 2-D Filtering / 3-D Reconstruction

Amplitudes ( $|F_{h,k}|$ )

5	10	7	11	5	4	10	5	6
6	6	21	9	3	8	1	13	9
4	14	8	8	5	3	1	5	1
9	16	15	7	8	11	7	4	5
9	14	19	10	85	18	25	14	10
10	10	13	15	46	27	15	18	7
8	6	7	7	12	6	14	10	6
9	5	6	6	21	16	8	3	6
6	3	5	7	15	13	4	13	5

Phases ( $\alpha_{h,k} / 10$ )

3	30	24	31	36	31	31	6	23
33	18	10	9	31	7	9	9	10
29	3	5	7	5	29	14	16	25
28	1	4	1	13	14	2	21	20
7	26	3	17	23	25	4	24	16
1	20	30	15	23	31	9	25	33
30	34	35	7	7	18	20	32	2
10	2	31	20	22	24	34	25	16
29	17	26	25	13	12	17	19	32

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 6. 2-D Filtering / 3-D Reconstruction

#### **Complete Fourier Averaging:**

- **All** unit cells are averaged
- A **single** structure factor is computed for each reciprocal lattice point
- Fourier synthesis of this reduced set of structure factors gives the reconstructed structure of a **single** unit cell

**Note:** Process is formally equivalent to performing filtering with mask holes of **infinitely small** diameter

*Will demo this next time in RobEM*

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.b Digital Processing Steps

#### 6. 2-D Filtering / 3-D Reconstruction

#### **3D Reconstruction:**

Diffraction phases and amplitudes (structure factors) measured at all points of **3D** reciprocal lattice by combining data from 2D diffraction patterns from many, independent views of the specimen

Rationale for collecting and combining information from different views depends in part on the type of specimen studied (*i.e.* its **symmetry**)

## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.c Hardware / Software

#### **Two Disadvantages of Digital Processing:**

Expense and complexity of required hardware and software

**Microdensitometer:** >\$100,000 for precision instrument

**Computers:** Relatively **cheap!**

For \$2,000-5,000 can now get reasonable compute power and storage capacity for single-user, interactive image processing environment

For ~\$75,000 can build a 32 node Beowolf cluster of PCs for computationally intensive calculations

**Software:** **Very expensive** in effort and cost (>>\$100,000) to write, test, and support a stable suite of programs for running image processing procedures



## III.D.3 Digital Fourier Analysis of Electron Micrographs

### III.D.3.c Hardware / Software

#### **Software:**

Many labs engaged in image processing develop 'in-house' software tailored to needs of specific research projects

Established, portable systems: several are available either commercially or for "free" (*e.g.* SPIDER, IMAGIC, EMAN, MDPP, SEMPER, etc.)

**Advantage:** may save considerable effort (and frustration) in the development and testing of programs

**Disadvantage:** strong possibility of being incorrectly implemented by "black-box", novice users

End of Sec.III.D.3